

**CORRECTIONS TO A RADICAL APPROACH TO REAL ANALYSIS, 2nd
EDITION**

page 11: paragraph 4, line 1: “Archimedes” should be “Archimedes’ ”

page 14, Exercise 2.1.1, in part b, the vertices should be at $(a, 1 - a^2)$, $(a + \delta, 1 - (a + \delta)^2)$, $(a + 2\delta, 1 - (a + 2\delta)^2)$

page 15: Exercise 2.1.2, last line, coordinates of first point should be $(k2^{-n}, 1 - k^22^{-2n})$.

page 15: Exercise 2.1.5, line (2.5), the second term should be $1/2^k$ rather than $1/(2^k n)$

page 16: Exercise 2.1.10.a, last line, “are all with” should read “are all within”

page 27: Exercise 2.3.11. This technique only works for $x \geq 4$. For $x = 2$ or 3 , find the square root of $1/2 = 1 - 1/2$ or $3/4 = 1 - 1/4$, respectively, then multiply your answer by 2.

page 28: Exercise 2.3.12. $(1 + x)^2$ should be $(1 + x)^a$.

page 47: line 1: “ $|x| < 1$ ” should be “ $0 < x < 1$ ”,

line 3: “If $|x|$ is larger than 1” should be “If x is larger than 1”.

page 54: The definition of C^p and analytic functions ignores a very real distinction between C^∞ functions and analytic functions. A function f that is an analytic function at x_0 must be C^∞ on an open interval containing x_0 , but more than that, there must be an open interval containing x_0 in which the power series at x_0 converges to f . The example given on page 55 is precisely an example of a function that is C^∞ for all x but is *not* analytic at $x = 0$.

page 56: Exercises 2.6.4 and 2.6.5. Delete the adjective “analytic.”

page 58: Figure 3.1, the label at the right endpoint of the interval should be x

page 59: line 2 should end with “. . . what do we mean by the”

Page 84: line 16, first term in the sequence should be $2/\pi$ rather than $1/\pi$.

Pages 86–7: starting at the second line above equation (3.43), the fact that $f(x_k)$ and $f(y_k)$ can be forced as close together as we wish by taking k sufficiently large relies on uniform continuity, which has not yet been established for continuous functions on closed

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bounded intervals. To avoid the need to use uniform continuity, replace the text starting at this point and continuing to the end of the proof with the following:

and $f(x_{k+1}), f(y_{k+1})$ lie on opposite sides of A .

Our sequences $x_1 \leq x_2 \leq \dots$ and $y_1 \geq y_2 \geq \dots$ satisfy the conditions of the nested interval principle and so there is a number c that lies in all of these intervals. Again, by the Archimedean definition of limit, we see that

$$\lim_{k \rightarrow \infty} x_k = \lim_{k \rightarrow \infty} y_k = c.$$

Since f is continuous at $x = c$, we know that

$$\lim_{k \rightarrow \infty} f(x_k) = \lim_{k \rightarrow \infty} f(y_k) = f(c).$$

Since a lies between $f(x_k)$ and $f(y_k)$ and each of these sequences has the common limit of $f(c)$, A must equal $f(c)$.

page 103, Exercise 3.4.6.f. By “decimal fraction” I mean a number in decimal form

page 103, Exercise 3.4.12, “increasing” should be “strictly increasing”

page 106: equation (3.52) third term of the series expansion for $F(x)$ should be $f'(a)(x - a)$ rather than $f(a)(x - a)$.

page 107: Theorem 3.11. The hypothesis should be that there is a neighborhood of $x = a$ in which all derivatives of f exist rather than just that all derivatives of f exist at $x = a$.

page 109: In “Definition: infinite limit and limit at infinity,” line 4 should read sufficiently close to a (but not equal to a). That is to say, there is a $\delta > 0$ so that $0 < |x - a| < \delta$ implies that

page 112, exercise 3.5.3, second displayed inequality, condition should read “if $0 < \alpha < 1$ ”

page 113, Exercise 3.5.8, last expression in displayed equation should be

$$\lim_{x \rightarrow 0} \frac{2x \sin(1/x) - \cos(1/x)}{1},$$

page 122: first paragraph following Theorem 4.1: ... I want to emphasize what it [omit is] does

page 127, 3rd line before exercises, change “The summands alternate” to “The signs of the summands alternate”

page 131: Corollary 4.8 (The Limit Ratio Test). Add: If the limit does not exist, then this test is inconclusive.

page 132: Corollary 4.10 (The Limit Root Test). Add: If the limit does not exist, then this test is inconclusive.

page 172: equation (5.5), in the limit after the equal sign, $\lim_{x \rightarrow 0}$ should be $\lim_{y \rightarrow 0}$

page 177, line 3 following "Rearrangement with Conditional Convergence," (5.13) should be (5.11)

page 193, Figure 5.7. There is no graph of $y = x$. Delete the phrase "with graph of $y = x$ included"

page 201: Exercise 5.3.3 should read: Prove that if $\sum g_k$ converges uniformly over the bounded interval I and if $f_k(x) = (x - a)g_k(x)$, then $\sum f_k$ converges uniformly over I .

page 202: Exercise 5.3.11. The reference should be to exercise 5.3.10, not 6.3.7.

pages 204–5, Proof of Corollary 5.13. There are problems with the proof as it stands. A correct and simpler proof is as follows: For fixed positive α strictly less than the radius of convergence, we know that the series converges absolutely at $x = \alpha$. Let $M_k = |a_k|\alpha^k$. For $|x| \leq \alpha$, $|a_k x^k| \leq M_k$. Therefore, we have uniform convergence by the Weierstrass M -test.

page 212: Exercise 5.4.1.b., summand should be $\frac{n^2}{\sqrt{n!}}(x^n + x^{-n})$, missing factorial in the denominator.

page 229: The proof of Lemma 6.3 contains an error. The fact that x is the upper limit of the x_n does not guarantee that $|x - x_n|$ can be made arbitrarily small for sufficiently large n . To see a corrected proof, go to www.maclester.edu/aratra/corrections/lemma6-3.pdf.

page 232: First displayed equation after (6.32) should be $|F(x+0) - f(x+2a)|$, missing closing parenthesis.

page 233: Definition of g in first displayed equation, top line should be $|F(x+0) - F(x+2u)|$, $0 < u \leq a$,

page 256, Last line of exercise 6.3.9, the functions f should be h .

page 259: Line 6. The proof in 1970 was by Joseph Gerver: The differentiability of the Riemann function at certain rational multiples of π . *American Journal of Mathematics*, **92**:1970, pp. 33–55.

page 274: Equation (A.8), the numerator of the fraction to the right of the equality should be $4 \cdot 6 \cdot 8 \cdots (2p + 2q - 2)$

page 279: first display after (A.21): upper limit of integration should be k [not 1]