The Dilogarithm

Appendix to *A Radical Approach to Real Analysis* 2nd edition

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The **dilogarithm** is defined for $-1 \le x \le 1$ by

$$\mathrm{Li}_2(x) = \sum_{k=1}^{\infty} \frac{x^k}{k^2}. \tag{1}$$

In general, the polylogarithm is defined by

$$\mathrm{Li}_n(x) = \sum_{k=1}^{\infty} \frac{x^k}{k^n}, \tag{2}$$

in the interval of convergence. The function $\mathrm{Li}_1(x)$ is related to the natural logarithm by

$$\mathrm{Li}_1(x) = \sum_{k=1}^{\infty} \frac{x^k}{k} = -\ln(1-x). \tag{3}$$

From equation (3), it is easy to see that

$$\mathrm{Li}_2(x) = \int_0^x \frac{-\ln(1-t)}{t}\, dt. \tag{4}$$

Equation (4) has the advantage that it is well-defined for all $x \le 1$.

There is a wealth of information on the dilogarithm on the *Wolfram MathWorld* site, http://mathworld.wolfram.com/Dilogarithm.html.