

Projects in Analysis
pdf files and links at
www.macalester.edu/~bressoud/talks

Rearranging the Harmonic Series:

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots = \ln 2.$$

$$1 + \frac{1}{3} - \frac{1}{2} + \frac{1}{5} + \frac{1}{7} - \frac{1}{4} + \dots = \frac{1}{2} \ln 2$$

$$\begin{aligned} & 1 + \frac{1}{3} + \dots + \frac{1}{2r-1} - \left(\frac{1}{2} + \frac{1}{4} + \dots + \frac{1}{2s} \right) \\ & + \left(\frac{1}{2r+1} + \frac{1}{2r+3} + \dots + \frac{1}{4r-1} \right) \\ & - \left(\frac{1}{2s+2} + \frac{1}{2s+4} + \dots + \frac{1}{4s} \right) + \dots \\ & = ? \end{aligned}$$

Inverse Symbolic Calculator at

www.cecm.sfu.ca/projects/ISC/ISCmain.html

proofs can be done using

$$1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} = \ln n + \gamma + E(n)$$

where $\lim_{n \rightarrow \infty} E(n) = 0$;

or by using the power series expansion of $\ln(1+x)$.

What does

$$\sqrt{2}^{\sqrt{2}^{\sqrt{2}^{\sqrt{2}^{\dots}}}}$$

mean? How large is it? For what values of x does

$$x^{x^{x^{x^{\dots}}}}$$

converge?

see Knoebel, “Exponentials reiterated”

Amer. Math. Monthly 88 (1981), no. 4, 235–252.

A sequence $\{x_n\}$ of real numbers is said to be **Cesàro convergent** to x_0 if the sequence of its averages has the limit x_0 .

$$\lim_{n \rightarrow \infty} \frac{x_1 + x_2 + \cdots + x_n}{n} = x_0.$$

Prove that if $\lim_{n \rightarrow \infty} x_n = x_0$, then the sequence $\{x_n\}$ is Cesàro convergent to x_0 .

We can use the symbol

$$x_n \xrightarrow{C} x_0$$

to mean that $\{x_n\}$ is Cesàro convergent to x_0 . We say that a function f is **Cesàro continuous** at $x = x_0$ if

$$x_n \xrightarrow{C} x_0 \quad \text{implies} \quad f(x_n) \xrightarrow{C} f(x_0).$$

Note that we have weakened the conclusion, but we have also weakened the hypothesis. Is every continuous function also Cesàro continuous? Is every Cesàro continuous function also continuous?