# Average Length of Stay in the Park

adapted from problem AB2/BC2 on the 2002 ©AP Calculus exam

The rate at which people enter an amusement park on a given day is modeled by the function $E$ defined by

$$E(t) = \frac{15600}{(t^2 - 24t + 160)}.$$

The rate at which people leave the same amusement park is modeled by the function $L$ defined by

$$L(t) = \frac{9890}{(t^2 - 38t + 370)}.$$

Both $E(t)$ and $L(t)$ are measured in people per hour and time $t$ is measured in hours after midnight. These functions are valid for $9 \leq t \leq 23$, the hours during which the park is open. At time $t = 9$, there are no people in the park.

**(a)** How many people have entered the park by 5 PM ($t = 17$)? Round answer to the nearest whole number.

**(b)** The price of admission to the park is \$15 until 5:00 PM ($t = 17$). After 5:00 PM, the price of admission to the park is \$11. How many dollars are collected from admissions to the park on the given day? Round your answer to the nearest whole number.

**(c)** Let $H(t) = \int_9^t (E(x) - L(x))\,dx$ for $9 \leq t \leq 23$. The value of $H(17)$ to the nearest whole number is 3725. Find the value of $H'(17)$ and explain the meaning of $H(17)$ and $H'(17)$ in the context of the park.

**(d)** At what time $t$, for $9 \leq t \leq 23$, does the model predict that the number of people in the park is a maximum?

**Other questions:**

**(e)** When is the rate at which people are entering the amusement park the greatest?

Ans. 12 noon

**(f)** At what time is the total number of people in the park increasing most quickly?

Ans. Between 11:41 and 11:42 AM ($t = 11.6918$)

**(g)** What is the greatest number of number of people in the park at any time?

Ans. 3951

**(h)** How many people are in the park when it closes at 11 PM?

Ans. 1

**(i) What is the average amount of time each visitor spends in the park?**

1. The first question to ask is "What is meant by the average amount of time each visitor spends in the park?". When one person spends one hour in the park, we will call that one person-hour. Three people, each spending two hours in the park or six people spending one hour apiece in the park, each creates six person-hours in the park. The average amount of time each visitor spends in the park is equal to

$$\frac{\text{the total number of person–hours spent in the park}}{\text{the total number of people who visit the park}}.$$

   Finding the value of the numerator will be tricky, but you should be able to find the value of the denominator.What is the total number of people who enter the park?

2. Let's start with a situation where it is a little easier to find the total number of person-hours spent in the park. If people enter the park at the rate given by $E(t)$, but everyone stays until 11 PM ($t = 23$), how many person-hours are spent in the park? The following analysis should help you find this answer.

   If we pick a time, say $t_i$, then the number of people who enter the park near this time is given by $E(t_i)\,\Delta t$. Actually, this is an approximation to the number of people who enter between $t_i - \Delta t/2$ and $t_i + \Delta t/2$. Each of these people stays in the park approximately $23 - t_i$ hours. For our simpler problem, the total number of person-hours spent in the park can be approximated by

$$\sum_i E(t_i)\,\Delta t\,(23 - t_i).$$

As we take smaller values of $\Delta t$, this approximating sum gets more accurate, and it also gets closer to the value of an integral. What is this integral? What is its value?

3. The answer to the previous question is clearly bigger than the number of person-hours spent in the park in our original problem because lots of people leave before 11 PM. If someone leaves at 3 PM, that decreases the total person-hours spent in the park by 8. The next paragraph will help you calculate how many person-hours are lost because people leave before 11 PM.

   We again pick a time $t_i$. The number of people who leave between time $t_i - \Delta t/2$ and time $t_i + \Delta t/2$ is approximated by $L(t_i)\Delta t$. Each of these people decreases the total number of person-hours spent in the park by $23 - t_i$ hours. Find the summation that approximates the number of person-hours that must be subtracted to account for people who leave before 11 PM.

4. Again, as $\Delta t$ approaches 0, this summation becomes more accurate, and it gets closer to the value of an integral. Find that integral and evaluate it.

5. What *is* the average amount of time each visitor spends in the park?

**Answers**

1. $\displaystyle\int_9^{23} E(t)\,dt \;=\; 7276$ people

2. $\displaystyle\int_9^{23} E(t)(23-t)\,dt \;=\; 66762$ person-hours

3. $\displaystyle\sum_i L(t_i)\,\Delta t\,(23-t_i)$. person-hours

4. $\displaystyle\int_9^{23} L(t)(23-t)\,dt \;=\; 36380$ person-hours

5. $\dfrac{66762-36380}{7276} = 4.176$ hours, approximately 4 hours and 11 minutes. (If you keep full accuracy in each intermediate result, you get 4.17603 hours. But the only reasonable answer would be "A little over four hours.")