Math 136, Fall '04

Project B: Planes in Space

The purpose of this project is to finish the analysis of the number of regions formed by n planes that was begun in the Polya video. First draft is due Friday, October 15. Final version is due Friday, November 5.

(1) Fill in the following table for the number of regions obtained when a solid, plane, or line is cut by n

n	planes	lines	points
0			
1			
2			
3			
4			
5			
6			

(2) Find a formula for the number of regions of a k-dimensional object cut by n objects. Hint: Express this number in terms of binomial coefficients $\binom{n}{k}$.

$n \backslash k$	0	1	2	3	4	5	6
0	1	0	0	0	0	0	0
1	1	1	0	0	0	0	0
2	1	2	1	0	0	0	0
3	1	3	3	1	0	0	0
4	1	4	6	4	1	0	0
5	1	5	10	10	5	1	0
6	1	6	15	20	15	6	1

(3) The challenge now is to *prove* that these formulas are valid for all n. Start with the easy one. Prove your formula for the number of line segments we get when we place n points on a line.

(4) Now tackle the proof for the formula for the number of regions that we get when we place n lines on a plane. If you are to prove this by induction, what do you need to show? How do you justify the inductive step that gets you from the formula with $n-1$ lines to the formula with n lines? Why should the number of regions that get added be equal to the number of line segements formed by putting $n-1$ points on a line? Think about what happens when that nth line gets added to the plane.

(5) You should now be ready to prove your formula for the number of regions formed by n planes. What do you need to show in order to establish an inductive proof? How do you justify the inductive step? Why is the number of 3-dimensional regions that get added equal to the number of regions in a plane cut by $n-1$ lines? Think about what happens as the nth plane cuts across the previous planes. What do their intersections look like?

(6) In 4-dimensional space, what is the formula for the number of 4-dimensional regions that are bounded by n 3-dimensional hyperplanes? What assumptions do you have to make about 3-dimensional hyperplanes in 4-dimensional space in order to prove your formula? Prove your formula.