First Project for Math 377, fall, 2003
Part 2: due Friday, October 31

1. When $r = 1$ and $s = 2$, the limiting value of `rearrange[n,1,2]` is $(1/2)\log 2$. Show that this is the value by assuming that the associative law holds so that you can group the first and second, fourth and fifth, seventh and eighth, etc. terms.

2. The following is a sketch of how you might convince someone that the limit of `rearrange[n,2,3]` is $\frac{1}{2}\log(8/3)$:

   Let

$$\begin{aligned} f(x) &= x^3 + \frac{x^9}{3} - \frac{x^4}{2} - \frac{x^8}{4} - \frac{x^{12}}{6} + \frac{x^{15}}{5} + \frac{x^{21}}{7} - \frac{x^{16}}{8} - \frac{x^{20}}{10} - \frac{x^{24}}{12} + \cdots \\ &= \frac{1}{2}\left[\log(1+x^3) - \log(1-x^3) + \log(1-x^4)\right] \\ &= \frac{1}{2}\log\left(\frac{(1+x^3)(1-x^4)}{1-x^3}\right). \end{aligned}$$

   Now take the limit as x approaches 1 from the left of $f(x)$. What is the limit of `rearrange[n,2,3]`?

3. In order to find the general formula for the limit of `rearrange[n,r,s]`, you want to assign powers of x to the summands so that:

   (a) The r positive summands and the s negative negative summands in the kth set all have exponents that are larger than the exponents of the terms in the $k-1$st set and that are smaller than the exponents of the terms in the $k+1$st set.

   (b) If you separate the positive terms from the negative terms, the exponents for the positive terms form an arithmetic sequence, and the exponents for the negative terms form an arithmetic sequence.

.

Second Assignment, due Friday, October 31, 25% of 1st project grade: Describe how you used the assumption that the associative law holds to suggest that the limit of `rearrange[n,1,2]` is $\frac{1}{2}\log 2$. Describe your process of filling in the missing steps in the argument that the limiting value of `rearrange[n,2,3]` is $\frac{1}{2}\log(8/3)$. Describe how you found the general formula. Note that these are suggestive or heuristic arguments, but not yet proofs. There are some assumptions that will need to be justified.

First Project for Math 377, fall, 2003
Part 3: due Friday, November 21

4. **Prove** that the general formula is correct. There are three parts to this. You have to prove that your series converges for $|x| < 1$. You have to prove that it converges at $x = 1$. And you have to prove that the series is a continuous function on $[0, 1]$.

   Final Assignment, due Friday, November 21, 50% of 1st project grade: Write a final project report that includes a complete statement and proof of the formula for the limiting value of `rearrange[r,s,iterations]`. This report should also summarize the key difficulties and insights that you encountered.