

Elab. 1.11

[Computation/intro/intro106]

Computer arithmetic is not exact for all numbers, particularly very large numbers and very small numbers. For example, 10^{50} is a very large number and, in computer arithmetic, is no different from $10^{50} + 1$. You can confirm this using an equality test:

```
> 1e50 == 1 + 1e50
[1] TRUE
```

Mathematically, the numbers on each side of the `==` are different, but in the computer arithmetic they are the same. So the result is mathematically incorrect.

Find the smallest exponent n that gives a mathematically **incorrect** result when comparing 10^n and $1 + 10^n$. (Remember that a number like 10^5 is written as `1e5`, not `10e5`.)

```
n = 10 11 12 13 14 15 16 17 18 19 20 Elab. 1.11-1
```

Similarly, very small numbers like 10^{-50} are effectively rounded to zero when they are added to much larger numbers. The statement $1 = 1 + 10^{-50}$ is mathematically false but gives a TRUE result on the computer. Find the smallest exponent m that gives a correct FALSE result in the test $1 = 1 + 10^{-m}$:

```
m = 10 11 12 13 14 15 16 17 18 19 20 Elab. 1.11-2
```