Activity 11.17 **[f2008/f2008-mg4]**

This exercise explores redundancy in $n = 3$ dimensional space. The general principles of redundancy are these:

- In n-dimensional space, it takes a minimum of n model vectors to be able to reach any potential target point.
- If there are more than n vectors, there will be redundancy. The extra vectors don't expand the set of target points that you could reach.
- But even when there are n vectors or even *fewer* than n vectors, there can be redundancy.

1) To start, create the following four vectors, a target and three model vectors, exactly as described:

```
> target = vec(4,1,-3)
> m1 = vec(2,5,1)
> m2 = vec(-3,5,0)
> m3 = vec(1,1,1)
```

As created, these three model vectors are not redundant. Here's a way to see that: create some new target points — any at all — and show that you can always reach an exact solution. For instance, try these target points:

```
> t1 = vec(0,1,7)
> t2 = vec(-3,2,4)
> t3 = vec(0,0,1)
```

For each of them, view the solution with `vsolve` (using the `B` key to speed things up). For instance:

```
> vsolve( mat(m1,m2,m3), t1)
```

2) Now, create a set of three model vectors with redundancy. You can do this by constructing one of the vectors to be a linear combination of the other two. For example, here is a new model vector that is a linear combination of `m1` and `m2`:

```
> m4 = 3*m1 + 2*m2
```

You could use any coefficients you want.

1. Fit the model using just `m1` and `m2`:

```
> vsolve( mat(m1,m2), target )
```

Use the B key to find the best solution.

- What is the length of the residual at the B solution?
- What is the relationship between the residual vector and the model vectors? (Hint: Use the c key to orient the display so that you are looking down one of the model vectors.)
- Use the x key to display the coefficients of the best solution.
- Quit vsolve using the q key, and compute the linear combination of m1 and m2 using the coefficients you found:

  ```
  > 0.71775*m1 - 0.60698*m2
  ```

  The result will be an $n = 3$-dimensional vector: the end-of-walk point.

2. Fit the model using just m1 and m4:

```
> vsolve( mat(m1,m4), target )
```

Use the B key to find the best solution.

- What is the length of the residual at the B solution?
- What is the relationship between the residual vector and the model vectors?
- Use the x key to display the coefficients of the B solution.
- Quit vsolve using the q key, and compute the linear combination of m1 and m4 using the coefficients you found. (Careful: make sure to use the same model vectors you gave to vsolve. It's easy to make a typing mistake.)
  How does the end-of-walk point here correspond to the end-of-walk point you got when you used m1 and m2 as model vectors?

3. Fit the model using the three model vectors m1, m2, and m4:

```
> vsolve( mat(m1,m2,m4), target )
```

Use the B key to find the best solution.

- What is the length of the residual at the B solution?
- What is the relationship between the residual vector and the model vectors?
- Again, use the x key to display the coefficients of the B solution. Quit vsolve and compute the end-of-walk point using those coefficients and the three model vectors. How does it compare to the point you found using m1 and m2 and to using m1 and m4?

3) Find a solution using all four model vectors: `m1`, `m2`, `m3`, and `m4`.

```
> vsolve( mat(m1,m2,m3,m4), target)
```

There's a redundancy in this set of four model vectors.

- Find a best solution using the `B` key. What is the length of the residual vector? Explain.
- Try to find another solution that is just as good. You'll have to do this by hand, since `B` will always find the same best solution.