Math 131/135/194, Fall 2004

Applied Calculus

Profs. Kaplan & Flath

Macalester College

Assignment 3: Units and Dimensions

1 Dimensional Analysis

Mathematical formulas describe a relationship between variables. The term dimensional analysis refers to examining the dimensions of these variables. Sometimes, just by looking at the variables' dimensions you can spot an error or improve your understanding of a relationship. For this assignment, the fundamental dimensions are time (T), mass (M), and length (L). Other dimensions can be derived from this. For example,

quantity	dimension	comments
velocity	L/T	
acceleration	LT^{-2}	
force	MLT^{-2}	Remember Newton's law: $F = ma$. A metric unit of force is the newton.
volume mass density	ML^{-3}	Mass per volume
linear mass density	ML^{-1}	Mass per length of something.
energy	ML^2T^{-2}	Energy is force multiplied by length. Typical units: joule, calories.
power	ML^2T^{-3}	Power is energy per unit time. Typical units: watt (joule/sec), horse power, BTU

All of this may seem very much like physics, but dimensional analysis applies in other areas as well.

Consider the area of your skin. Some physiological processes (heat radiation, perspiration) are proportional to skin area. Unlike your weight or height, it's not so easy to measure your skin area. However, there is a formula that provides an approximation.

$$A = \sqrt{w \times h/3600} \qquad (1)$$

where w is your weight[1] (in kg) and h is your height (in cm). The units of A will be in square-meters. This approximation can be useful in determining the dosage of certain classes of drugs. [2]

1. Calculate your own body-surface area. Is the result you get reasonable? In order to know this, you would have to have a good guess what your body-surface area actually is. Here are two ways to make a guess:
   - Imagine yourself as a cylinder. What would be the height h and radius r of a cylinder that is roughly the same size of your body? From these, you can calculate the area of the cylinder: $A = 2\pi r^2 + 2\pi rh$.
   - Model your or leg as a cylinder and calculate the surface area of that cylinder. Then apply a rule of thumb used by burn physicians in computing the fraction of skin that has been injured. This is called the "Rule of Nines" and divides the total surface area of the body into 11 units, each of which is 9% of the total skin area: each arm/hand, one unit 9%; each leg, two units, $2\times 9 = 18\%$, front of torso, two units, 18%; back of torso, two units 18%; head, one unit 9%. So, if you know the area of your arm and hand, divide that by 9% to get your total body surface area.
2. Show that the formula given in Equation 1 doesn't make dimensional sense. Keep in mind that the dimension of A should be L^2.
3. Suppose we re-write the formula slightly differently:

$$A = \sqrt{\frac{w \times h}{\alpha}} \qquad (2)$$

   We are using α to stand for a number *which might have some nontrivial dimension.* What would the dimension of α have to be in order for Equation 2 to make sense dimensionally?

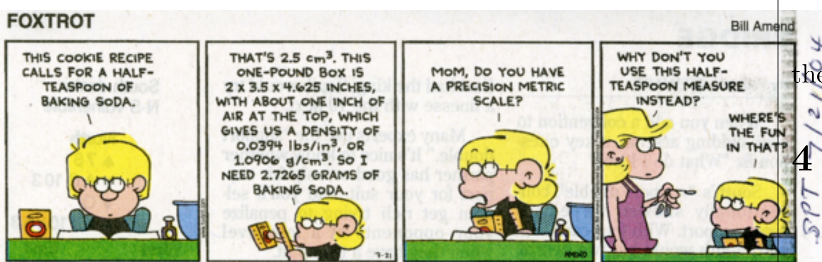
[1] Actually, the units of kg should serve as an indication that this is really your mass, not your weight. But the medical textbook from which this was drawn, and just about everyone else in the world other than scientists, considers kg to be a unit of weight. They are wrong.

[2] Anna M. Curren, Dimensional Analysis for Meds, 2nd ed. Delmar/Thomson, p. 189

4. In Equation 1, the number 3600 should have been written with some units. What do these units have to be in order for A to be in square meters? (Remember that w is in kg and h is in cm.)

5. Using simple unit conversion, find the appropriate value of α in Equation 2 if w is given in lbs, h in inches, and A is in square inches.

2 Unit Conversion



1. Is Foxtrot right?[3]

2. How much would the box weigh if it were filled with water? (The mass density of water is something that you should know from first principles in metric units ($\frac{1gm}{cc}$). In English units there is a similar relationship: one pint of water weighs one pound.)

3 Astronomy

We are to make a scale model of the Galilean solar system on campus. It has to fit in the playing field outside of our classroom, so the diameter of Saturn's orbit has to be 100 yards. Here's some data:

Body	Radius (km)	Dist. from Sun (10^6 km)
Sun	695,990	-
Mercury	2440	57.9
Venus	6054	108.2
Earth	6378	149.6
Moon	1738	
Mars	3396	227.9
Jupiter	71,492	778.3
Io	1821	
Europa	1565	
Ganymede	2634	
Callisto	2403	
Saturn	60,268	1429.4

How big does each of the planets and moons have to be in the scale model?

4 Conversion Problems

1. A new coal-burning power plant can generate 1 gigawatt (GW or billion watts) of power. Burning 1 kilogram of coal yields about 3 kilowatt-hours of energy. How much energy, in kilowatt-hours, can the plant generate each month? How much coal, in kilograms, is needed by this power plant each month? If a typical home uses 1000 kilowatt-hours per month, how many homes can this power plan supply with energy?[4]

2. In Europe, gasoline consumption is often described in terms of litres per 100 km. In the US, it is described in terms of miles per gallons. How many miles per gallon is 7 litres per 100 km?

   Aside: Kaplan's theory for the difference in descriptions is that in the US gasoline is relatively inexpensive, so people are mainly concerned with how far they can go with the amount of gas remaining in their tank. In Europe, gasoline is expensive, and it is important to know how much a given trip is going to cost. Miles per gallon is easily multiplied by gallons to give miles: the distance you can travel. Litres per 100 km is easily multiplied by km to give litres, which is proportional to expense.

[3]Thanks to Len Vacher for this problem.
[4]ibid., p. 116. In the original, it was stated that one kg of coal yields about 450 kilowatt-hours of electricity.