CHAPTER 1

What Is a Computation?

This book is about how to use computers to build your own specialized tools for carrying out work in science and technology. The computer, virtually unknown in its present form 50 years ago, by now has emerged fully as a universal tool of science and technology, as well as business, communication, and even entertainment. The computer has achieved this universal status in large part because it is a general-purpose tool that can be configured to perform specialized tasks. We accomplish this configuration most often by writing software: instructions that tell the computer how to be the tool we want it to be.

1.1 Computation as Transformation

Much of the reason why the computer is so useful and adaptable is that it is an *information tool*. We are all familiar with physical tools like screwdrivers and hammers that amplify and concentrate forces and thereby allow us to change the configuration of physical things. Similarly, an information tool is one that allows us to change the configuration of information. This may be hard to understand because "information" itself is such an elusive concept.

Let us use this idea of transforming information to offer a definition of a computation:

*A **computation** is a transformation from one or more inputs to an output.*

This definition doesn't specify what is an input or an output, but perhaps you can anticipate that both of them are information in some form.

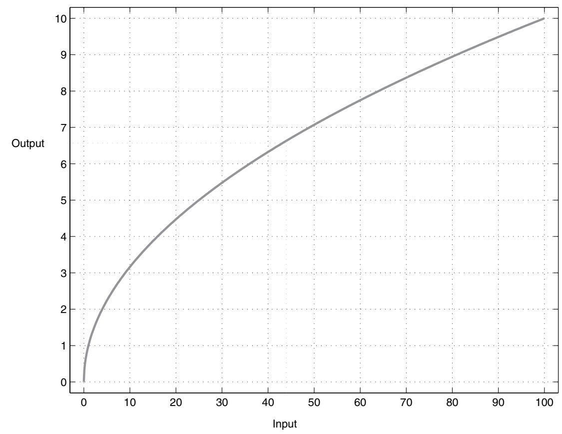


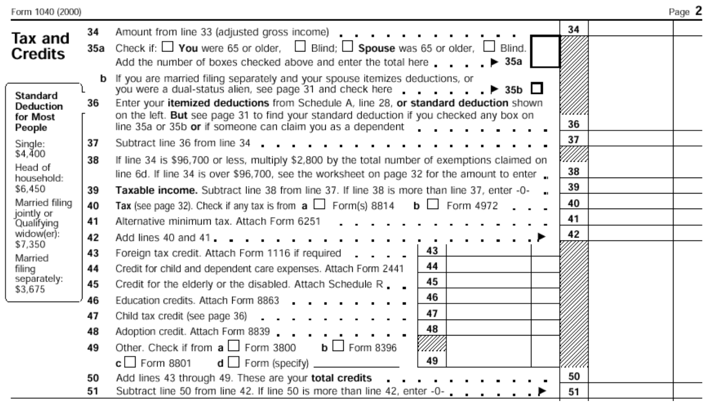
Figure 1.1. A graphical means of computing square roots.

We are concerned in this book with a particular modern technology for carrying out computations: software for the electronic computer. You will see many examples of such computations in the coming chapters. But for now, let us emphasize that computations are carried out all the time and everywhere by myriad other means than electronics. Some of these computations have evolved naturally and others involve human technologies of various sorts. The following are some noncomputerized examples of computations.

The graph in Figure 1.1 shows an ordinary plot of $y = \sqrt{x}$. The graph is part of a simple technology for carrying out a computation, finding the square root of a number. The input is x; the output is y. We all learn how to use such graphs to carry out a transformation from x to y: (1) Given the numerical value of x, find the corresponding point on the x-axis. (2) Trace a vertical line from this point up to intersect the curve. (3) From the intersection point, trace a horizontal line to meet the y-axis, and (4) read off the position of the meeting point on the y-axis. For example, an input of $x = 44$ leads to an output of $y \approx 6.6$.

Each year, U.S. citizens are required to compute their taxes. The inputs are numbers representing income, the number and ages of children, and so on. The output is the amount of tax owed. The government provides a detailed description of the algorithm to be followed in the shape of the form shown in Figure 1.2. Actually, the government does not require that its citizens do the computation; they are obliged only to provide the input information. The government will carry out the computation based on the inputs and send the output in the form of a bill or refund check.

Look at Figure 1.3. It's likely that a specific name occurs to you. If not, probably you recognize the picture as that of a woman or a face. There is a computation involved here too. The input is the collection of small squares



Form 1040 (2000) Page 2

Tax and Credits

Standard Deduction for Most People
Single: $4,400
Head of household: $6,450
Married filing jointly or Qualifying widow(er): $7,350
Married filing separately: $3,675

34 Amount from line 33 (adjusted gross income) . . . 34
35a Check if: ☐ **You** were 65 or older, ☐ Blind; ☐ **Spouse** was 65 or older, ☐ Blind.
Add the number of boxes checked above and enter the total here . . . ▶ 35a
b If you are married filing separately and your spouse itemizes deductions, or you were a dual-status alien, see page 31 and check here . . . ▶ 35b ☐
36 Enter your **itemized deductions** from Schedule A, line 28, **or standard deduction** shown on the left. **But** see page 31 to find your standard deduction if you checked any box on line 35a or 35b **or** if someone can claim you as a dependent . . . 36
37 Subtract line 36 from line 34 . . . 37
38 If line 34 is $96,700 or less, multiply $2,800 by the total number of exemptions claimed on line 6d. If line 34 is over $96,700, see the worksheet on page 32 for the amount to enter . 38
39 **Taxable income.** Subtract line 38 from line 37. If line 38 is more than line 37, enter -0- . 39
40 **Tax** (see page 32). Check if any tax is from **a** ☐ Form(s) 8814 **b** ☐ Form 4972 . . . 40
41 Alternative minimum tax. Attach Form 6251 . . . 41
42 Add lines 40 and 41 . . . ▶ 42
43 Foreign tax credit. Attach Form 1116 if required . . . 43
44 Credit for child and dependent care expenses. Attach Form 2441 44
45 Credit for the elderly or the disabled. Attach Schedule R . . 45
46 Education credits. Attach Form 8863 . . . 46
47 Child tax credit (see page 36) . . . 47
48 Adoption credit. Attach Form 8839 . . . 48
49 Other. Check if from **a** ☐ Form 3800 **b** ☐ Form 8396 **c** ☐ Form 8801 **d** ☐ Form (specify) 49
50 Add lines 43 through 49. These are your **total credits** . . . 50
51 Subtract line 50 from line 42. If line 50 is more than line 42, enter -0- . . . ▶ 51

Figure 1.2. The algorithm for computing income taxes for individuals in the United States.

(pixels) of various shades of gray. The output is some other representation of the input's information: perhaps "Mona Lisa" or "woman" or "face."

The algorithm by which this transformation is performed is by no means completely known, although computers have been taught to compare a picture of a face to a database of such pictures.

Figure 1.4 shows graphs of short segments of speech recorded by a microphone. If you were to play these sounds one at a time through a speaker, you would hear EE for the top sound and OH for the bottom sound. The way that humans accomplish this transformation from sound input to an output as a perceived vowel is not unique. Unlike image recognition, computerized speech recognition is within the range of today's technology; successful computer algorithms have been developed for carrying out a sound-to-perception computation that can translate sound to text.

Figure 1.3. A low-resolution image consisting of 1008 pixels.

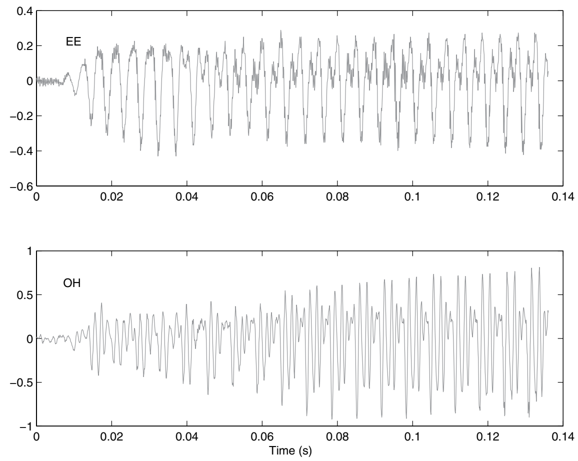


Figure 1.4. Speech sounds as recorded by a microphone. Top: EE. Bottom: OH. The signals are in the files `ee.wav` and `oh.wav`.

1.2 Computation as Reaction to Events

The definition of a computation as a transformation from inputs to outputs ignores the issue of where the inputs come from. These inputs often come from measurements.

A measurement (for instance the length of an object, the pH of a chemical solution, the electrical voltage difference between two points in a circuit, the time between heart beats, or the IQ of a test subject) assigns a value, often numerical, to some object or event. As such, the measurement is a translation from an input—the object or event—to an output; a measurement is itself a computation.

Key Term

Measurements are performed using sensors or *transducers*.[1] A mercury thermometer, for example, is a transducer from temperature to position of the mercury meniscus. A speedometer transduces vehicle speed into the position of a pointer. In both cases, the numerical value of the measurement can be read from a graduated scale. Even a computer keyboard is a kind of transducer, measuring which key is pressed at what time.

Some of the earliest computational devices were designed to transduce from the physical alignment of the earth and the sun to a reading of time. The antique sundial shown in Figure 1.5 is considerably more complicated than the decorative variety found in gardens today. The complication stems from the need for accurate measurement of time in the days before high-

[1]The word *transduce* is from the Latin for "across" (trans) and "lead" (ducere). It is closely related to the word *translate,* which comes from "across" and "carried" (latus).