Name Key Section Fall 2002

Experiment 11

Advance Study Assignment: The Atomic Spectrum of Hydrogen

1. The helium ion, He^+, has energy levels similar to those of the hydrogen atom, since both species have only one electron. The energy levels of the He^+ ion are given by the equation

$$E_n = -\frac{5248.16}{n^2} \text{ kJ/mole} \quad n = 1, 2, 3, \ldots$$

a. Calculate the energies in kJ/mole for the four lowest energy levels of the He^+ ion.

eg for $n=1$, $E_1 = -\frac{5248.16}{1^2} \text{ kJ mol}^{-1}$

$E_1 =$ -5248.16 kJ/mole

$E_2 =$ -1312.04 kJ/mole

$E_3 =$ -583.13 kJ/mole

$E_4 =$ -328.01 kJ/mole

$E_2 - E_1 = -1312.04 \frac{kJ}{mol} - \left(-5248.16 \frac{kJ}{mol}\right)$
$= 3936.12 \text{ kJ/mol}$

$\Delta E = h\nu = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{\Delta E}$

b. One of the most important transitions for the He^+ ion involves a jump from the $n = 2$ to the $n = 1$ level. ΔE for this transition equals $E_2 - E_1$, where these two energies are obtained as in Part a. Find the value of ΔE in kJ/mole. Find the wavelength in nm of the line emitted when this transition occurs; use Equation 4 to make the calculation.

$$\lambda = \left(\frac{6.626\times10^{-34}\,\cancel{J}\cdot\cancel{s}}{\text{particle}}\right)\left(\frac{2.998\times10^{8}\,\cancel{m}}{\cancel{s}}\right)\left(\frac{\text{mol}}{3936.12\,\cancel{kJ}}\right) \times \left(\frac{6.023\times10^{23}\,\cancel{\text{particle}}}{\cancel{\text{mol}}}\right)\left(\frac{\cancel{kJ}}{10^3\,\cancel{J}}\right)\left(\frac{10^9\,\text{nm}}{\cancel{m}}\right) = 30.40\text{ nm}$$

$\Delta E =$ 3936.12 kJ/mole; $\lambda =$ 30.40 nm

c. Three of the strongest lines in the He^+ ion spectrum are observed at the following wavelengths: (1) 121.57 nm; (2) 164.12 nm; (3) 468.90 nm. Find the quantum numbers of the initial and final states for the transitions that give rise to these three lines. Do this by calculating, using Equation 4, the wavelengths of lines that can originate from transitions involving any two of the four lowest levels. You calculated one such wavelength in Part b. Make similar calculations with the other possible pairs of levels. When a calculated wavelength matches an observed one, write down n_{hi} and n_{lo} for that line. Continue until you have assigned all three of the lines.

Let's derive a general expression for λ as a function of n_{hi} and n_{lo}:

(1) 4 → 2 (2) 3 → 2 (3) 4 → 3

$$\Delta E = -5248.16 \frac{kJ}{mol}\left(\frac{1}{n_{hi}^2} - \frac{1}{n_{lo}^2}\right) = \frac{hc}{\lambda} \Rightarrow \lambda = -\frac{hc}{5248.16 \text{ kJ/mol}}\left(\frac{1}{n_{hi}^2} - \frac{1}{n_{lo}^2}\right)^{-1}$$

$$\lambda = -\left(\frac{6.626\times10^{-34}\,\cancel{J}\cdot\cancel{s}}{\cancel{\text{part}}}\right)\left(\frac{2.998\times10^{8}\,\cancel{m}}{\cancel{s}}\right)\left(\frac{\cancel{kJ}}{10^3\,\cancel{J}}\right)\left(\frac{10^9\,\text{nm}}{\cancel{m}}\right)\left(\frac{6.023\times10^{23}\,\cancel{\text{part}}}{\cancel{\text{mol}}}\right)\left(\frac{\cancel{\text{mol}}}{5248.16\,\cancel{kJ}}\right) \times \left(\frac{1}{n_{hi}^2} - \frac{1}{n_{lo}^2}\right)^{-1}$$

$$\lambda = -22.80\text{ nm}\left(\frac{1}{n_{hi}^2} - \frac{1}{n_{lo}^2}\right)^{-1}$$

Now try $n_{hi} = 4 \Rightarrow n_{lo} = 3, 2, 1$: 4→3 is 469 nm, 4→2 is 122 nm

$n_{hi} = 3 \Rightarrow n_{lo} = 2, 1$: 3→2 is 164 nm