Recasting the $2p_{+1}$ and $2p_{-1}$ orbitals into a more useful form:

$$\psi_{2p_{+1}} = R_{21}(r)\, Y_{11}(\theta,\phi)$$

$$= \left[\frac{1}{4\sqrt{6}}\left(\frac{Z}{a_0}\right)^{3/2}\left(\frac{2Zr}{a_0}\right)e^{-Zr/2a_0}\right]\left[-\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{i\phi}\right]$$

$$= \underbrace{\left[\frac{1}{4\sqrt{6}}\left(\frac{Z}{a_0}\right)^{3/2}\left(\frac{2Z}{a_0}\right)\left(-\sqrt{\frac{3}{8\pi}}\right)e^{-Zr/2a_0}\right]}_{-f(r)} r\sin\theta\, e^{i\phi}$$

$$\psi_{2p_{+1}} = -[f(r)]\, r\sin\theta\, e^{i\phi}$$

Similarly, $\psi_{2p_{-1}} = R_{21}(r)\, Y_{1-1}(\theta,\phi) = [f(r)]\, r\sin\theta\, e^{-i\phi}$

As already proven, I am free to take any linear (aka superposition) combination of degenerate energy eigenfunctions I want, with the guarantee that this linear combination will also be an energy eigenfunction (i.e., it will satisfy the Schrodinger equation).

Let me choose

$$\psi = -\frac{1}{\sqrt{2}}\psi_{2p_{+1}} + \frac{1}{\sqrt{2}}\psi_{2p_{-1}}$$

Choosing $C_1 = -\frac{1}{\sqrt{2}}$ and $C_2 = +\frac{1}{\sqrt{2}}$ will

(a) ensure my new ψ is normalized

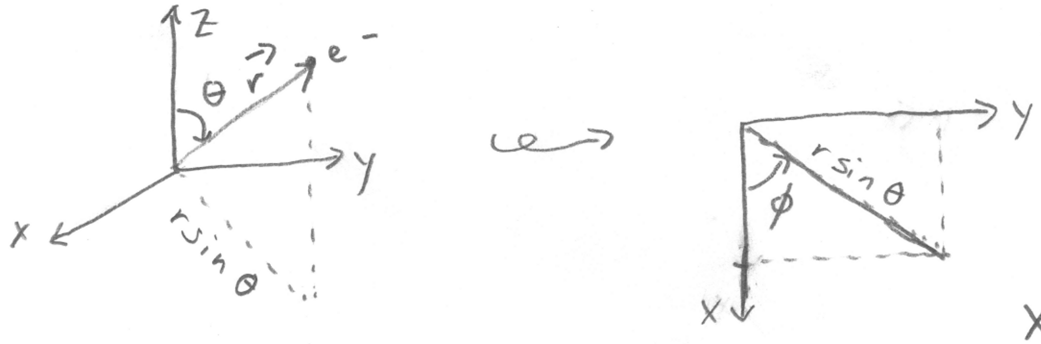
(b) get rid of the imaginary part of the spherical harmonics

$$\psi = \frac{1}{\sqrt{2}}[f(r)]\, r\sin\theta\, e^{i\phi} + \frac{1}{\sqrt{2}}[f(r)]\, r\sin\theta\, e^{-i\phi}$$

$$\psi = \frac{1}{\sqrt{2}}[f(r)]\, r\sin\theta\, [e^{i\phi} + e^{-i\phi}]$$

[and since $e^{i\phi} = \cos\phi + i\sin\phi$
and $e^{-i\phi} = \cos\phi - i\sin\phi$,
$e^{i\phi} + e^{-i\phi} = 2\cos\phi$]

$$\psi = \frac{2}{\sqrt{2}}[f(r)]\, \underbrace{r\sin\theta\cos\phi}_{\text{and what is this in Cartesian coordinates?}}$$



$$\cos\phi = \frac{x}{r\sin\theta}$$
$$\Downarrow$$
$$\underline{x = r\sin\theta\cos\phi}$$

so $\psi = -\frac{1}{\sqrt{2}}\psi_{2p_{+1}} + \frac{1}{\sqrt{2}}\psi_{2p_{-1}} = \sqrt{2}[f(r)]\, x$

and $\psi = 0$ when $x = 0$. This eigenfunction has an angular node in the yz plane, and is concentrated on the x-axis.

$$\therefore \boxed{-\frac{1}{\sqrt{2}}\psi_{2p_{+1}} + \frac{1}{\sqrt{2}}\psi_{2p_{-1}} \equiv \psi_{2p_x}}$$ (node at $x=0$)

Similar arguments would show

$$\boxed{\frac{i}{\sqrt{2}}\psi_{2p_{+1}} + \frac{i}{\sqrt{2}}\psi_{2p_{-1}} \equiv \psi_{2p_y}}$$ (node at $y=0$)