

### Chapter 4 Example Problems

Note the rules for significant figures when we do calculations with uncertainties:

- (1) The last significant figure occurs in the same decimal place as the uncertainty.
- (2) You should always write a number and its uncertainty to the same accuracy.
- (3) You may report an uncertainty to more than one digit (if, for example, you want to minimize rounding error), but you must write these additional non-significant figures as subscripts to the right of the decimal place.

1. In Minnesota, you are considered legally drunk if your blood alcohol content (bac) is greater than 0.100 weight % ethanol (EtOH). A Breathalyzer used by the state police has a well-established  $\sigma = 0.006\%$  EtOH. An erratically-driving St. Thomas student is pulled over and his bac is tested. One Breathalyzer measurement gives a reading of 0.107%. Is he in trouble at (a) the 68.3% confidence level? (b) At the 95.5% confidence level?
2. Kowalski's wants to sell the Hog Brew brand of beer. However, under Minnesota state law, grocery stores cannot sell any beverage containing more than 3.2 weight % alcohol. The store's resident analytical chemist uses gas chromatography on a bottle of Hog's Brew and makes the following measurements (all in wt. %): 3.24, 3.27, 3.32, 3.28. For these data,  $\bar{x} = 3.27_8$  and  $s = 0.03_3$ . Can Kowalski's legally sell Hog Brew?
3. In Spring 2002, Macalester students Pamela Peralta Yahya and Omar Zgheib determined the phosphorous content in both name-brand (Enfamil) and generic infant formulas. (Their concern was that low-income families, who could afford only the generic product, were being shortchanged on nutrients like phosphorous.) Three measurements were made on both formulas. Ironically, Enfamil was found to contain  $57.6 \pm 0.4$  mg P/serving, while the generic was found to contain  $59.4 \pm 0.4$  mg P/serving. (The uncertainty stated is one standard deviation). Determine if the two brands contain significantly different amounts of P at the 95% confidence level.
4. To determine how well Analytical Chemistry students washed their  $\text{Fe}(\text{OH})_3 \cdot x\text{H}_2\text{O}$  with hot 1%  $\text{NH}_4\text{NO}_3$ , a team of Olin-Rice 2nd-floor scientists and a team of Olin-Rice 3rd-floor scientists measure the amount of  $\text{Cl}^-$  still remaining in four samples:

mg $\text{Cl}^-$ still in analyte			
Sample	Team Bio	Team Chem	$d$
A	0.150	0.145	+0.005
B	0.280	0.275	+0.005
C	0.100	0.095	+0.005
D	0.650	0.644	+0.006

$$\bar{d} = 0.0052_5 \quad s_d = 0.0005_0$$

Is there a significant difference in the two teams' results at the 95% confidence level?

5. Kowalski's still has its heart set on selling Hog Brew. It consults two distinguished chemists to perform analyses on alcohol content. Their results are summarized here:

wt % EtOH			
Brand	KTK	TDV	$d$
Hog Brew	3.28	3.19	0.09
Samuel Adams	3.39	3.40	-0.01
Bud Light	2.79	2.73	0.06
Beck's	3.97	3.88	0.09

$$\bar{d} = 0.05_8$$

$$s_d = 0.04_7$$

Is there a significant difference between KTK's and TDV's results at the 95% confidence level?

6. X-ray diffraction is a powerful method for determining the structure of crystals. Analysis of a set of measurements yields a quantity  $A$  that allows one to calculate the distance between adjacent atoms in a crystal. In principle,  $A$  should be constant for all measurements. Here are  $A$  values from analysis of a sample of tungsten:

700      625      575      603      587      599      605      593

Can any of these data be rejected at the 95% confidence level?

### Q-values at various confidence levels (Rorabacher, D.B. *Analytical Chemistry* 1991, 63, 139.)

Table I. Critical Values of Dixon's  $r_{10}$  ( $Q$ ) Parameter As Applied to a Two-Tailed Test at Various Confidence Levels, Including the 95% Confidence Level<sup>a</sup>

$N^b$	confidence level					
	80% ( $\alpha = 0.20$ )	90% ( $\alpha = 0.10$ )	95% ( $\alpha = 0.05$ )	96% ( $\alpha = 0.04$ )	98% ( $\alpha = 0.02$ )	99% ( $\alpha = 0.01$ )
3	0.886	0.941	<b>0.970</b>	0.976	0.988	0.994
4	0.679	0.765	<b>0.829</b>	0.846	0.889	0.926
5	0.557	0.642	<b>0.710</b>	0.729	0.780	0.821
6	0.482	0.560	<b>0.625</b>	0.644	0.698	0.740
7	0.434	0.507	<b>0.568</b>	0.586	0.637	0.680
8	0.399	0.468	<b>0.526</b>	0.543	0.590	0.634
9	0.370	0.437	<b>0.493</b>	0.510	0.555	0.598
10	0.349	0.412	<b>0.466</b>	0.483	0.527	0.568
11	0.332	0.392	<b>0.444</b>	0.460	0.502	0.542
12	0.318	0.376	<b>0.426</b>	0.441	0.482	0.522
13	0.305	0.361	<b>0.410</b>	0.425	0.465	0.503
14	0.294	0.349	<b>0.396</b>	0.411	0.450	0.488
15	0.285	0.338	<b>0.384</b>	0.399	0.438	0.475
16	0.277	0.329	<b>0.374</b>	0.388	0.426	0.463
17	0.269	0.320	<b>0.365</b>	0.379	0.416	0.452
18	0.263	0.313	<b>0.356</b>	0.370	0.407	0.442
19	0.258	0.306	<b>0.349</b>	0.363	0.398	0.433
20	0.252	0.300	<b>0.342</b>	0.356	0.391	0.425
21	0.247	0.295	<b>0.337</b>	0.350	0.384	0.418
22	0.242	0.290	<b>0.331</b>	0.344	0.378	0.411
23	0.238	0.285	<b>0.326</b>	0.338	0.372	0.404
24	0.234	0.281	<b>0.321</b>	0.333	0.367	0.399
25	0.230	0.277	<b>0.317</b>	0.329	0.362	0.393
29	0.227	0.273	<b>0.312</b>	0.324	0.357	0.388
27	0.224	0.269	<b>0.308</b>	0.320	0.353	0.384
28	0.220	0.266	<b>0.305</b>	0.316	0.349	0.380
29	0.218	0.263	<b>0.301</b>	0.312	0.345	0.376
30	0.215	0.260	<b>0.298</b>	0.309	0.341	0.372

<sup>a</sup>In this and the other accompanying tables, the newly generated or corrected values are indicated in boldface. <sup>b</sup>Sample size.