Chapter 4 Example Problems

Note the rules for significant figures when we do calculations with uncertainties:

(1) The last significant figure occurs in the same decimal place as the uncertainty.
(2) You should always write a number and its uncertainty to the same accuracy.
(3) You may report an uncertainty to more than one digit (if, for example, you want to minimize rounding error), but you must write these additional non-significant figures as subscripts to the right of the decimal place.

1. In Minnesota, you are considered legally drunk if your blood alcohol content (bac) is greater than 0.100 weight % ethanol (EtOH). A Breathalyzer used by the state police has a well-established σ = 0.006% EtOH. An erratically-driving St. Thomas student is pulled over and his bac is tested. One Breathalyzer measurement gives a reading of 0.107%. Is he in trouble at (a) the 68.3% confidence level? (b) At the 95.5% confidence level?

2. Kowalski's wants to sell the Hog Brew brand of beer. However, under Minnesota state law, grocery stores cannot sell any beverage containing more than 3.20 weight % alcohol. The store's resident analytical chemist uses gas chromatography on a bottle of Hog's Brew and makes the following measurements (all in wt. %): 3.24, 3.27, 3.32, 3.28. For these data, $\bar{x} = 3.27_8$ and $s = 0.03_3$. Can Kowalski's legally sell Hog Brew?

3. In Spring 2002, Macalester students Pamela Peralta Yahya and Omar Zgheib determined the phosphorous content in both name-brand (Enfamil) and generic infant formulas. (Their concern was that low-income families, who could afford only the generic product, were being shortchanged on nutrients like phosphorous.) Three measurements were made on both formulas. Ironically, Enfamil was found to contain 57.6 ± 0.4 mg P/serving, while the generic was found to contain 59.4 ± 0.4 mg P/serving. (The uncertainty stated is one standard deviation). Determine if the two brands contain significantly different amounts of P at the 95% confidence level.

4. To determine how well Analytical Chemistry students washed their $Fe(OH)_3 \cdot xH_2O$ with hot 1% NH_4NO_3, a team of Olin-Rice 2nd-floor scientists and a team of Olin-Rice 3rd-floor scientists measure the amount of Cl^- still remaining in four samples:

mg Cl^- still in analyte

Sample	**Team Bio**	**Team Chem**	***d***
A	0.150	0.145	+0.005
B	0.280	0.275	+0.005
C	0.100	0.095	+0.005
D	0.650	0.644	+0.006

$\bar{d} = 0.0052_5$ $\qquad s_d = 0.0005_0$

Is there a significant difference in the two teams' results at the 95% confidence level?

5. Kowalski's still has its heart set on selling Hog Brew. It consults two distinguished chemists to perform analyses on alcohol content. Their results are summarized here:

wt % EtOH

Brand	**KTK**	**TDV**	***d***
Hog Brew	3.28	3.19	0.09
Samuel Adams	3.39	3.40	-0.01
Bud Light	2.79	2.73	0.06
Beck's	3.97	3.88	0.09

$\bar{d} = 0.05_8$ $s_d = 0.04_7$

Is there a significant difference between KTK's and TDV's results at the 95% confidence level?

6. X-ray diffraction is a powerful method for determining the structure of crystals. Analysis of a set of measurements yields a quantity *A* that allows one to calculate the distance between adjacent atoms in a crystal. In principle, *A* should be constant for all measurements. Here are *A* values from analysis of a sample of tungsten:

700 625 575 603 587 599 605 593

Can any of these data be rejected at the 95% confidence level?

Q-values at various confidence levels (Rorabacher, D.B. *Analytical Chemistry* **1991**, *63*, 139.)

Table I. Critical Values of Dixon's r_{10} (Q) Parameter As Applied to a Two-Tailed Test at Various Confidence Levels, Including the 95% Confidence Level[a]

	confidence level					
N[b]	80% (α = 0.20)	90% (α = 0.10)	95% (α = 0.05)	96% (α = 0.04)	98% (α = 0.02)	99% (α = 0.01)
3	0.886	0.941	**0.970**	0.976	0.988	0.994
4	0.679	0.765	**0.829**	0.846	0.889	0.926
5	0.557	0.642	**0.710**	0.729	0.780	0.821
6	0.482	0.560	**0.625**	0.644	0.698	0.740
7	0.434	0.507	**0.568**	0.586	0.637	0.680
8	0.399	0.468	**0.526**	0.543	0.590	0.634
9	0.370	0.437	**0.493**	0.510	0.555	0.598
10	0.349	0.412	**0.466**	0.483	0.527	0.568
11	0.332	0.392	**0.444**	0.460	0.502	0.542
12	0.318	0.376	**0.426**	0.441	0.482	0.522
13	0.305	0.361	**0.410**	0.425	0.465	0.503
14	0.294	0.349	**0.396**	0.411	0.450	0.488
15	0.285	0.338	**0.384**	0.399	0.438	0.475
16	0.277	0.329	**0.374**	0.388	0.426	0.463
17	0.269	0.320	**0.365**	0.379	0.416	0.452
18	0.263	0.313	**0.356**	0.370	0.407	0.442
19	0.258	0.306	**0.349**	0.363	0.398	0.433
20	0.252	0.300	**0.342**	0.356	0.391	0.425
21	0.247	0.295	**0.337**	0.350	0.384	0.418
22	0.242	0.290	**0.331**	0.344	0.378	0.411
23	0.238	0.285	**0.326**	0.338	0.372	0.404
24	0.234	0.281	**0.321**	0.333	0.367	0.399
25	0.230	0.277	**0.317**	0.329	0.362	0.393
29	0.227	0.273	**0.312**	0.324	0.357	0.388
27	0.224	0.269	**0.308**	0.320	0.353	0.384
28	0.220	0.266	**0.305**	0.316	0.349	0.380
29	0.218	0.263	**0.301**	0.312	0.345	0.376
30	0.215	0.260	**0.298**	0.309	0.341	0.372

[a] In this and the other accompanying tables, the newly generated or corrected values are indicated in boldface. [b] Sample size.