From Atkins and de Paula's *Physical Chemistry*, 8th ed.

2.3(b) A sample consisting of 2.00 mol He is expanded isothermally at 22°C from 22.8 dm^3 to 31.7 dm^3 (a) reversibly, (b) against a constant external pressure equal to the final pressure of the gas, and (c) freely (against zero external pressure). For the three processes calculate q, w, ΔU, and ΔH.

In all 3 cases, the initial states are the same and the final states are the same. ∴ ΔU (and ΔH, which we'll discuss later) must also be the same.

Specifically, $\boxed{\Delta U = n C_{v,m} \Delta T = 0}$ since $\Delta T = 0$

(a) Reversibly is when $p_{ext} = p_{sys}$ (more in a moment)

$$\text{so } w = -nRT \ln\left(\frac{V_f}{V_i}\right) = -(2.00\ \text{mol})\left(\frac{8.31447\ \text{J}}{\text{mol K}}\right)(295.15\ \text{K}) \times \ln\left(\frac{31.7\ \text{dm}^3}{22.8\ \text{dm}^3}\right) = \boxed{-1.617\ \text{kJ}}$$

and $\boxed{q = -w = +1.617\ \text{kJ}}$

(b) $p_{ext} = p_f$ throughout expansion,

$$\text{and } p_f = \frac{nRT}{V_f} = (2.00\ \text{mol})\left(\frac{0.082058\ \text{L atm}}{\text{mol K}}\right)\left(\frac{295.15\ \text{K}}{31.7\ \text{L}}\right) = 1.528\ \text{atm}$$

$$\text{so } w = -p_{ext}\Delta V = (-1.528\ \text{atm})\underbrace{(31.7 - 22.8)}_{\text{down to 2 sf!}}\text{L}\left(\frac{101.325\ \text{J}}{\text{L atm}}\right)$$

$\boxed{w = -1.378\ \text{kJ}} \Rightarrow \boxed{q = +1.378\ \text{kJ}}$