Chap. 1 Example Problem (Haber Process)

Calculations

(a) $pV = nRT$ or $\frac{p}{T} = \frac{nR}{V}$ = constant $= \frac{P_2}{T_2} = \frac{P_1}{T_1}$

$400.°C + 273.15 \Rightarrow 673._{15}$ K
$-50.°C + 273.15 \Rightarrow 223._{15}$ K
$29°C + 273.15 \Rightarrow 302._{15}$ K

or $P_2 = P_1\left(\frac{T_2}{T_1}\right)$

When $T_2 = 400.°C$,

$$P_2 = (30.0\,\text{atm})\left(\frac{673._{15}\,\cancel{K}}{302._{15}\,\cancel{K}}\right) = \boxed{66.8_4\,\text{atm}}$$

When $T_2 = -50.°C$

$$P_2 = (30.0\,\text{atm})\left(\frac{223._{15}\,\cancel{K}}{302._{15}\,\cancel{K}}\right) = \boxed{22.1_6\,\text{atm}}$$

(b) At 400.°C, % error $= \frac{66.8_4\,\cancel{atm} - 68.9\,\cancel{atm}}{68.9\,\cancel{atm}} \times 100\% = \boxed{-3.0\%}$

At -50.°C, % error $= \frac{22.1_6\,\cancel{atm} - 21.5\,\cancel{atm}}{21.5\,\cancel{atm}} \times 100\% = \boxed{+3.1\%}$

(c) vdw: $p = \frac{nRT}{V-nb} - a\left(\frac{n}{V}\right)^2$

Need n. Calculate by assuming N_2 behaves as a perfect gas at 29°C.

$$n = \frac{pV}{RT} = \frac{(30.0\,\text{atm})(500.\,\text{L})}{(0.082058\,\text{L atm mol}^{-1}\text{K}^{-1})}\left(\frac{1}{302._{15}\,\text{K}}\right) = \underline{604._{99}\,\text{mol}}$$

At 400.°C,

$$P_{vdw} = \frac{(604._{99}\,\text{mol})(0.082058\,\text{L atm mol}^{-1}\text{K}^{-1})(673._{15}\,\text{K})}{500.\,\text{L} - (604._{99}\,\text{mol})(3.87\times10^{-2}\,\text{L mol}^{-1})} - \left(1.352\,\frac{\text{atm L}^2}{\text{mol}^2}\right)\left[\frac{(604._{99})^2\,\text{mol}^2}{(500.)^2\,\text{L}^2}\right]$$

[N.B. (1) $1\,\text{dm}^3 \equiv 1\,\text{L}$
(2) $V-nb$ here equals $476._6$ L, or ~5% lower than V]

$P_{vdw} = 70.1_2\,\text{atm} - 1.97_9\,\text{atm} = \boxed{68.1_4\,\text{atm}} > P_{ideal}$

⇑ > P_{ideal} (very close to exp)

At -50.°C,

$$P_{vdw} = \frac{(604._{99}\,\text{mol})(0.082058\,\text{L atm mol}^{-1}\text{K}^{-1})(223._{15}\,\text{K})}{500.\,\text{L} - (604._{99}\,\text{mol})(3.87\times10^{-2}\,\text{L mol}^{-1})} - \left(1.352\,\frac{\text{atm L}^2}{\text{mol}^2}\right)\left[\frac{(604._{99})^2\,\text{mol}^2}{(500.)^2\,\text{L}^2}\right]$$

$= 23.2_4\,\text{atm} - 1.97_9\,\text{atm} = \boxed{21.2_6\,\text{atm}} < P_{ideal}$

⇑ > P_{ideal} (again) (again, very close to exp.)

(d) Calculate compression factors for N_2 based on the experimental pressures:

At 400.°C,

$$Z \equiv \frac{pV}{nRT} = \frac{(68.9\ \text{atm})(500.\ \text{L})}{(604.99\ \text{mol})(8.2058\times10^{-2}\ \text{L atm mol}^{-1}\text{K}^{-1})(673.15\ \text{K})}$$

$$\boxed{Z = 1.03_1} > 1 \Rightarrow \text{net intermolecular forces are } \underline{\text{repulsive}}$$

At −50.°C,

$$Z \equiv \frac{pV}{nRT} = \frac{(21.5\ \text{atm})(500.\ \text{L})}{(604.99\ \text{mol})(8.2058\times10^{-2}\ \text{L atm mol}^{-1}\text{K}^{-1})(223.15\ \text{K})}$$

$$\boxed{Z = 0.970_4} < 1 \Rightarrow \text{net intermolecular forces are } \underline{\text{attractive}}$$