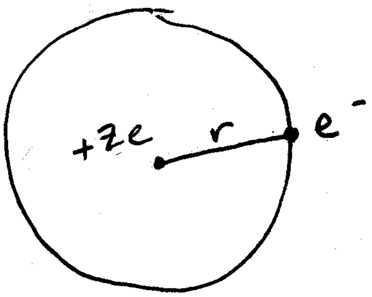
The de Broglie Model and the Atom



Potential energy (E_p) says

$$\boxed{E_p = -\frac{Ze^2}{4\pi\varepsilon_0 r}}$$

($\varepsilon_0 = 8.854 \times 10^{-12}\ C^2 J^{-1} m^{-1}$)

As $r \to 0$, $E_p \to -\infty$

So why doesn't e^- collapse into nucleus?

Consider kinetic energy (E_K) in terms of the e^-'s wave properties:

- $E_K = \frac{1}{2} m v^2$ (by definition) [1]
- $\lambda = \frac{h}{mv}$ (de Broglie wavelength)

  $\Downarrow$

  $v = \frac{h}{m\lambda}$ [2]
- $\lambda = \frac{L}{n'}$, $n' = 1, 2, 3, \ldots$ (circular standing wave) [3]

[2] into [1]:

$$E_K = \frac{1}{2}\cancel{m}\left(\frac{h^2}{m^{\cancel{2}}\lambda^2}\right) = \frac{h^2}{2m\lambda^2} \quad [4]$$

[3] into [4]:

$$E_K = \frac{h^2}{2m^{\cancel{2}}}\left(\frac{n'}{L}\right)^2 = \frac{h^2(n')^2}{2mL^2} \qquad [5]$$

Remember that n' = # of wavelengths that fit along the circumference of a circular orbit

Last time, we defined n = # of <u>half</u>-wavelengths along a standing wave

$$\therefore\ n = 2n' \Rightarrow n' = \frac{n}{2} \qquad [6]$$

Why care about n?

(1) Still true that # of nodes $= n - 1$

(2) # of nodes is chemically significant

[6] into [5]:

$$E_K = \frac{h^2\left(\frac{n}{2}\right)^2}{2mL^2} = \boxed{\frac{h^2n^2}{8mL^2}} \quad \text{where } n = 2, 4, 6, \ldots$$

for a circular standing wave

Say we confined e^- to an orbit only as big as the nucleus. How fast would we be forcing the e^- to move?

$$E_K = \frac{1}{\cancel{2}_{1}} m v^2 = \frac{h^2 n^2}{\cancel{8}_{4}\, m L^2}$$

$$v^2 = \frac{h^2 n^2}{4 m^2 L^2} \Rightarrow \underline{v = \frac{h n}{2 m L}}$$

$$L = 2\pi r = 2\pi\,(\text{radius of nucleus})$$

$$\cong 2\pi\,(1 \times 10^{-15}\,\text{m}) = 6 \times 10^{-15}\,\text{m}$$

Assume lowest-energy oscillation, that is, $n = 2$.

$$\text{Then } v = \frac{(6.6 \times 10^{-34}\,\cancel{J}\cdot\cancel{s})(\cancel{2})}{\cancel{2}(9.1 \times 10^{-31}\,\cancel{kg})(6 \times 10^{-15}\,\cancel{m})} \left(\frac{\cancel{kg}\, m^{\cancel{2}}\, s^{-\cancel{2}\,1}}{\cancel{J}}\right)$$

$$\underline{v = 1 \times 10^{11}\ \text{m s}^{-1}}$$

- So, where is the electron?

  Wherever the electron wave is oscillating ... simultaneously!

  - the only thing we know for sure is where the e^- cannot be found (i.e. at a node)
  - the fact that an e^- is a wave means that it is delocalized ... it cannot be pinned down to any one spot

What's Wrong with de Broglie?

⊖ Still can't predict the properties of atoms/ions with more than 1 e^-...

∴ need a three-dimensional version of de Broglie's theory, that is, quantum mechanics