Econ 361: Intermediate Microeconomics
Prof. Sarah West
Homework 3
30 points

1. (5 points) Construct a worksheet in Excel using the information provided below showing quantity combinations for two goods (good *A* and good *B*) providing a consumer identical levels of satisfaction (Utility), and information about prices and income. Then answer the questions below.[1]

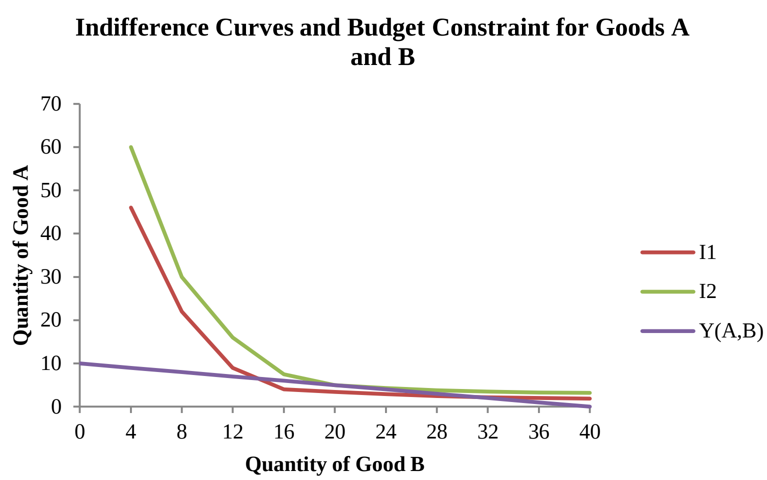
a. Construct a line chart for U_1 and U_2 using the given data for $Q(A)@ U_1$ and $Q(A)@ U_2$. Use the quantity data for good *B* as the x-axis.

See part c.

b. Are these well-behaved indifference curves? Explain.

The curves are convex, do not cross, and they are consistent with the assumption that more is better, as I_2 contains the same amount of good B for a given amount of good A than I_1.

c. Now add the budget constraint line to your chart. Print out your line chart from Excel and staple it to your homework.



d. At what quantities of good A and good B will the consumer attain his or her maximum utility given the budget constraint (line) *Y(A,B)*? Explain.

Utility is maximized at Q(B) = 20 and Q(A) = 5, because this is the only market basket he can attain with this budget constraint that allows him to reach I_2.

[1] Material for this problem was taken from Duke, et al. (2004) *Microsoft Excel for Microeconomics* (Upper Saddle River Pearson, Prentice Hall).

2. (5 points) For an individual the substitution effect of a change in the price of x on *x* equals -4, and the change in *x* with respect to a change in income equals -3. Under what condition will *x* be a Giffen good for this person? Use the Slutsky equation in your answer.

A Giffen good is an inferior good with an upward sloping demand curve, as the magnitude of the income effect of a price change is greater than the magnitude of the substitution effect. Formally, dX/dPx >0.

The Slutsky equation says:

$$dX/dPx = dx/Px|u=u^* + (dX/dI)(dI/dPx)$$

or,

The total change in X with respect to a change in price = substitution effect + income effect.

We know the substitution effect, -4, and dX/dI = -3, and dX/dPx must be positive. So,

$$(-4) + (-3)*(dI/dPx) > 0$$

$$-3*dI/dPx > 4$$

$$dI/dPx < -4/3$$

The change in income with respect to a change in price must be less than (greater in magnitude than) -4/3. This will yield a positive value for dX/dPx.

Intuitively, this is saying that the effect of a price change on purchasing power must be larger than -4/3. For people to exhibit Giffen behavior, the good in question must represent a large proportion of their total expenditure.

3. (5 points) Mary's utility function is given by $U_R = I^{1/4}$ and Dave's utility function is given by $U_J = I^{1/3}$, where *I* is income. For incomes greater than 100, who is more risk averse? Explain.

Mary is willing to pay a higher risk premium than Dave to avoid a given amount of risk because she is more risk averse. Specifically, the difference between a) the expected value of a risky scenario and b) the guaranteed income that yields the same utility as the expected utility of the risky scenario will be greater for Mary than for Dave. This difference is the risk premium.

Assume both are choosing between a guaranteed income of I on the one hand, and an uncertain income that has a 50% probability of being (I+a) and a 50% probability of being (I-a)

The expected value of risky scenario is .5(I+a) + .5(I-a) = I

The expected utility of the risky scenario is, for Mary:

$E(Utility)_{Mary}$ = (I+a)^1/4 + (I-a)^1/4

And for Dave:

$E(Utility)_{Dave}$ =(I+a)^1/3 + (I-a)^1/3

Comparing the two, we can see that $E(Utility)_{Mary} < E(Utility)_{Dave.}$

The guaranteed income (I_g) that yields the same utility as the expected utility above is, for Mary:

$E(Utility)_{Mary}$ = I^(1/3)
I_g = E(Utility)^3

And for Dave:

$E(Utility)_{Dave}$ = I^(1/4)
I_g = E(Utility)^4

From this we can see that Mary requires a lower guaranteed income than Dave. Recalling that the risk premium is defined as:

$RP = EV - I_g$

And Mary requires a lower guaranteed income, then her RP will be larger for a given gamble. *She is willing to pay more to avoid a given amount of risk than Dave is, because we know she is willing to accept a lower level of guaranteed income in order to avoid that risk.*

For these particular functions, the relationship switches for Income values of less than 1. For incomes greater than 100, Mary's maximum RP is larger, and is more risk-averse.

4. (5 points) Raymond and Sarah are the only consumers in a market for Star Wars wrist watches. Strangely, but lucky for you, despite the fact that they are the only two buyers of these watches, they are both price takers.

Sarah's utility is a function of her consumption of these watches, x, and of a composite of all other goods, y. Her utility function is given by: $U_S = x^{1/4}y^{3/4}$.

Raymond's utility is a function of his consumption of these watches, x, and of a composite of all other goods, y. His utility function is given by: $U_R = (1/5) \log x + (4/5) \log y$.

What is the market demand for watches, X? Derive the equation and draw the graph.

First, derive the demand of X from Sarah:

$d\varphi/dX = (1/4)X^{(-3/4)}*Y^{(3/4)} - \lambda Px = 0$
$d\varphi/dY = (3/4)Y^{(-1/4)}*X^{(1/4)} - \lambda Py = 0$
$d\varphi/d\lambda = PxX + PyY - I = 0$

Rearranging and dividing the first equation by the second yields:

$(d\varphi/dX) / (d\varphi/dY) = Y/3X = Px/ Py$

$PyY = 3X*Px$

Substituting into the budget constraint yields:

$I = PxX + 3X*Px$
$I = 4*PxX$
<u>$Xs = I/(4*Px)$</u> and this is the demand for X from Sarah.

Now, derive the demand from Raymond:

$\varphi = (1/5)\log(x) + (4/5)\log(y) - \lambda[PxX + PyY - I]$

$d\varphi/dX = (1/5)/X - \lambda Px = 0$
$d\varphi/dY = (4/5)/Y - \lambda Py = 0$
$d\varphi/d\lambda = PxX + PyY - I = 0$

$(d\varphi/dX) / (d\varphi/dY) = ((1/5)/X)/ ((4/5)/Y) = \lambda Px/ \lambda Py$

$Y/4X = Px/Py$

At this point, we can see that this is the same equation as before. From here on out the results are identical.

$PyY = 4X*Px$
$PxX = (1/4)Y*Py$

$I = PxX + 4X*Px$
$I = 5*PxX$
<u>$Xr = I/(5*Px)$</u>

As they are the only two buyers of these watches, the demand of X is the sum of Xs and Xr;
$X = 9I/(20*Px)$

5. (5 points) You are a financial planner. On behalf of each of your clients you can invest in one of two investments, which have the following payoffs and probabilities associated with each payoff. If your goal is to make your clients happy, for what kinds of clients would you recommend Investment A? Investment B? Be sure to use specific numerical justifications in your answers.

Payoff	Probability (Investment A)	Probability (Investment B)
$600	0.30	0.10
$500	0.40	0.80
$400	0.30	0.10

The expected value for Investment A is 0.30*(600) + 0.40*(500) + 0.30*(400) = $500
The expected value for Investment B is 0.10*(600) + 0.80*(500) + 0.10*(400) = $500

Var(A) = 0.3*(600-500)^2 + 0.4*(500-500)^2 + 0.3*(500-400)^2 = 6000
Var(B) = 0.1*(600-500)^2 + 0.8*(500-500)^2 + 0.1*(500-400)^2 = 2000

Both investments yield the same expected return, however, the variability of B is much larger than that of A. Thus, the recommendations depend on the client's preference towards risk. If the client is a risk-lover, I would recommend Investment A, and Investment B for the more risk averse client.

6. (5 points) Suppose that Kathy is currently earning an income of $51,000 (I = 51) and can earn that income next year with certainty. Her utility function is given by $U = (9I)^{1/2}$.

a. Kathy is offered a chance to take a new job that offers a .5 probability of earning $75,000 (I = 75) and a .5 probability of earning $40,000 (I = 40). Should she take the new job? Use numerical calculations in your explanation.

The expected utility of the new job: 0.5*(9*75)^0.5 + 0.5*(9*40)^0.5 = 22.48

The utility of her current job: (9*51)^0.5 = 21.42

Since 22.48 > 21.42, the expected utility of her new job is higher than the utility of her current job. Kathy will take the new job.

b. Would Kathy be willing to buy insurance to protect against the variable income associated with the new job? If so, how much would she be willing to pay for that insurance? Use numerical calculations in your answer.

Yes. She will be willing to buy insurance.

Her willingness to pay for insurance equals the amount of her risk premium. The risk premium is the amount Kathy would be willing to pay so that she receives the expected salary for certain rather than the risky salary in her new job.

In part b, we determined that her new job has an expected utility of 22.48. We need to find the certain salary that gives Kathy the same utility of 22.48, so we want to find I such that U(I) = 22.48.

Using her utility function, we want to solve the following equation: $\sqrt{9I} = 22.48$ which gives us I = 56.15. So Kathy would be equally happy with a certain salary of $56.15K or the uncertain salary with an expected value of $57.5K. Her risk premium is $57.5 - $56.15 = $1.349K. Kathy would be willing to pay $1349.95 to guarantee her income would be $57.5 thousand for certain and eliminate the risk associated with her new job.