**Homework 5**
**Econ 361**
**Intermediate Microeconomic Analysis**
**Spring 2012**
**30 points**

**1. (5 points) Do the following functions exhibit increasing, constant, or decreasing returns to scale? What happens to the marginal product of each individual factor as that factor is increased and the other factor held constant?**

**a.** $q = L + 2K$

Returns to scale are constant:
2q = 3*(2L) + 2*(2K)
2q = 2*(L+2K)

MPL is constant (at 1) and MPK is constant (at 2).

**b.** $q = L^{1/5} K^{1/2}$

Returns to scale are decreasing:
$2q > (2L)^{1/5} (2K)^{1/2}$
$2q > 2^{1/5+1/2} (L^{1/5} K^{1/2})$
$2q > 1.624*(L^{1/5} K^{1/2})$

MPL is diminishing (*d*MPL/*d*L < 0), MPK is diminishing (*d*MPK /*d*K < 0).

**c.** $q = L^{2/3}K^{2/3}$

Returns to scale are increasing:

$2q < (2L)^{2/3}(2K)^{2/3}$
$2q < 2.52*(L^{2/3}K^{2/3})$

MPL is diminishing (*d*MPL/*d*L < 0), MPK is diminishing (*d*MPK /*d*K < 0).

**d.** $q = 3K^{1/4} + 5L$

Returns to scale are decreasing:

$2q > 5(2L)+3(2K)^{1/4}$
$2q > 5(2L)+(2^{1/4})*3K^{1/4}$
$2q > 10L + 3.568*K^{1/4}$

MPL is diminishing (*d*MPL/*d*L < 0), MPK is constant (*d*MPK /dK = 0).

**2. (5 points) Consider a production function given by q = $L^{2/3}K^{2/3}$ (where *L* is labor and *K* is capital and a total cost function given by TC = $q^2$. Under what condition or conditions could these functions belong to the same firm? Explain.**

They belong to the same firm, when both functions exhibit both increasing returns to scale and diseconomies of scale.

With q=L^⅔ K^⅔, the firm is experiencing increasing returns to scale (see the previous question for how to show this).

TC=q^2 shows that as q doubles, it requires more than doubles of total cost. This is diseconomies of scale.

So the point to have them happen at the same time:

TC=wL+rK. When the price to labor "w" and the price to capital "r" increase in a way that overweights the increase in input's effect on total output, then the firm could experience both q=L^⅔ K^⅔, increasing returns to scale and diseconomies of scale at the same time.

Or, when the firm experience something else, such as input price rises due to finite resources, technology breaks down when too much output is given and the effects outweighs the IRS effect.

**3. (5 points) A firm's output is given by $q = 10K^{1/2}L^{1/2}$. Its capital, *K*, is fixed and equals 100 units per hour. The wage paid per worker-hour (*w* paid per unit *L*) is $15, and the cost per unit of capital per hour is $50. If worker-hours *L* rise from 9 to 16, by how much will marginal cost rise?**

MPL = $5K^{1/2}L^{-1/2}$
K = 100
Thus, MPL = $50\ L^{-1/2}$

MC = w/MPL

W = 15

When L = 9, MC = 15/(50/3) = 0.9
When L = 16, MC = 15/(50/4) = 1.2
So if worker-hours rise from 9 to 16, MC will rise by 1.2 – 0.9 = 0.3

**4. (5 points) My cousin is considering opening a lawn mowing business. He hires you as a consultant, and asks you to help him think through how to structure his business.**

> **a. What kind of production function is my cousin likely to have in this business? Explain the relationship between inputs and output.**

Answers will vary.

If we raise the number of lawnmowers per worker, we increase output per worker. That is because the more lawnmowers we have, the more backup we have in case a lawn mower breaks down. More lawnmowers per worker means less time that workers have to spend idle while waiting for a working lawnmower. The properties of returns to scale and substitution depend on the characteristics of the function.

To show diminishing returns, we can use a function with an exponent that is less than one. For example, we could have a production function given by:

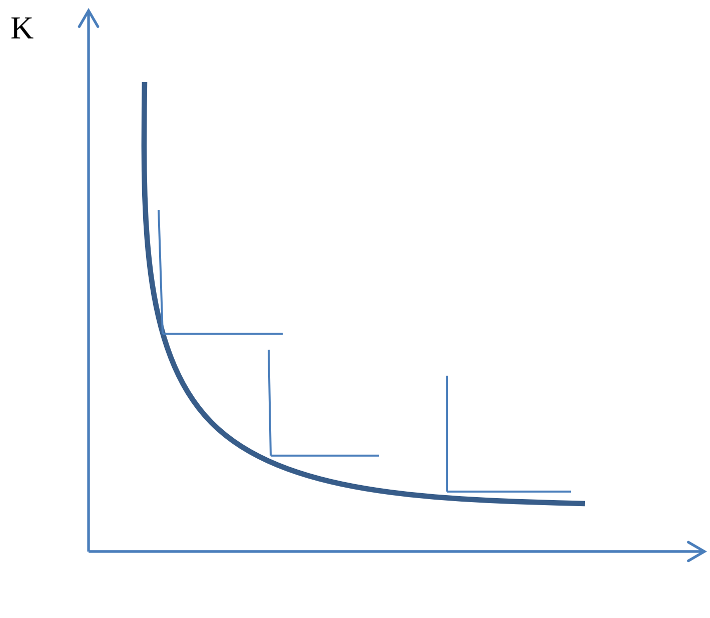
$q = K^{1/3}L^{2/3}$

Suppose this is the production function. If we double both of the inputs,

$q = K^{1/3}L^{2/3}$
$q_2 = (2K)^{1/3}(2L)^{2/3}$
$q_2 = (2^{1/3})*(2^{2/3})* K^{1/3}L^{2/3}$
$q_2 = (2^{1/3})*(2^{2/3})*q$
$(2^{1/3})*(2^{2/3}) = 2$
$q_2 = 2q$

So, doubling inputs would scale output by or 2, implying that this production function is characterized by constant returns to scale.

Now, with the production function above, there is some degree of substitutability between labor and capital. It may be the case that conditional on the availability of a specific kind of capital, labor and capital are perfect complements. But clearly in class we saw that there is a range of possible ways in which capital and labor can be combined. That is, it is possible to substitute labor for capital broadly speaking, but the inputs act as perfect complements conditional on a specific level of capital being available OR conditional on a given set of input prices that make a specific level of capital optimal:




So my cousin needs to first evaluate what the available capital is and what the relative prices of capital and labor are. Then, after making his optimal choice of levels, he may be able to conceptualize inputs as perfect complements (though it is difficult to accept that another unit of labor would have zero marginal product unless accompanied by more capital).

> **b. Given this production function, how should my cousin go about choosing the number of workers (per hour) and the number of lawnmowers (per hour) that he should use? Explain.**

Given the production function, he should choose to minimize costs subject to attaining specific level of output. At the optimum combination of workers and lawnmowers,

What you would actually say to him is that he should use inputs so that the additional amount of lawn mowed per dollar spent on the input should be equal for all inputs. To solve,

Slope of Isocost = MPL/MPK = MRTS = w/r

In this case,
MPL = $2/3\ K^{2/3}L^{-1/3}$
MPK = $1/3\ K^{-2/3}L^{2/3}$

So MPL/MPK = 2K/L = w/r
So K = wL/2r

$q = K^{1/3}L^{2/3}$
$= (wL/2r)^{1/3}L^{2/3}$
$= (w/2r)^{1/3}L$

So, $L = q\ (2r/w)^{1/3}$

$K = q\ (w/2r)^{2/3}$

*Alternatively, we can approach this question with Lagrangians Technique. The steps to answering this question are found in the appendix to chapter 7.*

Set up the Lagrangian:

$\Phi = wL + rK - \lambda[K^{1/3}L^{2/3}\text{-}q]$

Differentiate with respect to L, K, and λ:

1) $d\Phi/dL = w - \lambda[2/3K^{2/3}L^{-1/3}] = 0$
2) $d\Phi/dK = r - \lambda[1/3K^{-2/3}L^{2/3}] = 0$
3) $d\Phi/d\lambda = K^{1/3}L^{2/3} – q = 0$

Solve 1) for w and 2) for r

$w = \lambda[2/3K^{2/3}L^{-1/3}]$
$r = \lambda[1/3K^{-2/3}L^{2/3}]$

Solve for r/w:

$r/w = .5L/K$

Solve for L and K:

$L = 2rK/w$
$K =.5Lw/r$

Substituting these back into 3) yields L and K as functions of q, r, and w:

$q = (.5Lw/r)^{1/3}L^{2/3}$
$q = L*(.5w/r)^{1/3}$
$\mathbf{L = q/(.5w/r)^{1/3}}$

$q = K^{1/3}(2rK/w\ )^{2/3}$
$q = K*(2r/w)^{2/3}$
$\mathbf{K = q/(2r/w)^{2/3}}$

**5. (3 points) Assume a firm uses two inputs, labor and capital. If the marginal product of capital divided by the user cost of capital is less than the marginal**

**product of labor divided by the wage, then to minimize costs should this firm use more labor and less capital?**

The answer to this question is uncertain.

If the firm is not already using only labor, then it should use more labor and less capital. If, as described, MPK/r < MPL/w, then increasing L and decreasing K until the equality holds will increase output without increasing costs. In other words, if the *output per dollar of a unit of an additional unit of labor* is greater than *the output per dollar of a unit of an additional unit of capital*, then reallocating dollars from capital to labor will increase output without changing total costs.

Alternatively, we can think about this as reducing costs for a given level of output. Either way, reducing K and increasing L is the correct strategy.

However, if this is already using only labor then the firm cannot add any further labor (it has already arrived at a corner solution).

**6. (3 points) If for a firm total costs more than double as output doubles, then is the firm is experiencing decreasing returns to scale?**

Possibly, but not necessarily. This is the definition of diseconomies of scale, so the question is asking whether diseconomies of scale are always caused by decreasing returns to scale. Recall that returns to scale deal with how proportional changes in inputs proportionally change output (the easiest way to think of this is whether doubling inputs more or less than double output). Decreasing returns to scale could lead to diseconomies of scale if decreasing inputs had no effect on input prices (factors markets are perfectly competitive). But this might not be the case, as factor prices could go down.

More importantly, diseconomies of scale might be caused by something entirely different. If a firm actually has constant or increasing returns to scale, but as it purchases fewer inputs in factor markets it has little market power and will be charged for higher factor prices, this can cause diseconomies of scale then.

Returns to scale and economies of scale are different concepts, and while decreasing returns to scale can be the cause of diseconomies of scale, it often is not.

**7. (4 points) Find a real world firm that exhibits decreasing returns to scale, and one that exhibits increasing returns to scale. Provide the name of the firm and evidence that supports your designation of the firm as one with decreasing or increasing returns to scale. Cite your sources.**

(1) For DRS,
Wiles (1961) found that decreasing return to scale is rare in real life. An example on Shengyi Technology Company from Guangdong, China is found. As the

company experienced rapid growth in first half 2010 with more aluminum production, research analysts are concerned with the potential of DRS as the approaching of optimum production point. Major indications include the increase of input and production costs as hurdles for production.

Source:
Wiles, P.J. *Price, Cost, and Output,* p.213. New York: Fraeger, 1961.

Example retrieved online from
http://stock.eastmoney.com/news/1406,20100831932734 23.html

(2) For IRS
An example where this is the case would be Annheuser-Busch or Molson Coors, which are major brewers of beer. Because they produce output in volume (beer) but capital inputs in surface area (tanks, pipes, vats), doubling this capital and the labor needed to operate it would more than double output. To see why, consider the case of increasing a tank's size from 100 square feet of steel to 200 square feet of steel. That increase will more than double the amount of beer that tank holds, because a volume function is cubic while surface area is only raised to the second power.

Source: Intermediate Microeconomics And its Applications By Walter Nicholson, Christopher Snyder