

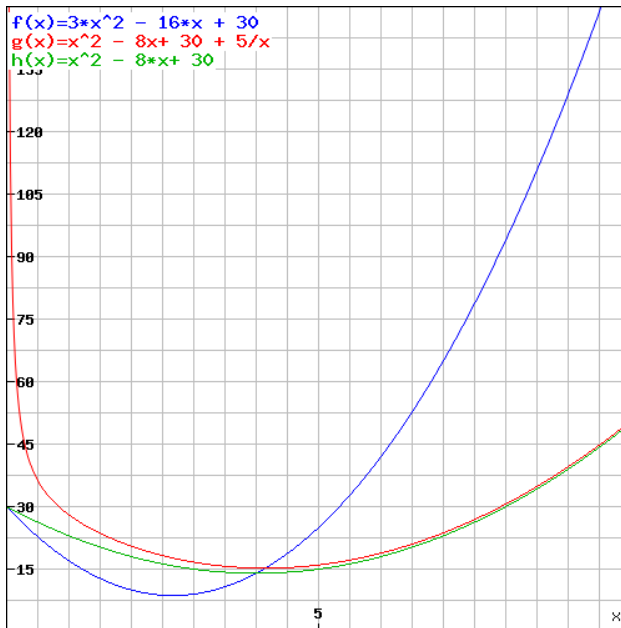
**Homework 6 Solutions**  
**Econ 361**  
**Intermediate Microeconomic Analysis**  
**30 points**

1. (4 points) A competitive firm has the following short-run cost function:

$$C(q) = q^3 - 8q^2 + 30q + 5$$

a. Find MC, ATC, and AVC and sketch them on a graph.

$$\begin{aligned} MC &= 3q^2 - 16q + 30 \\ ATC &= q^2 - 8q + 30 + 5/q \\ AVC &= q^2 - 8q + 30 \end{aligned}$$



b. At what range of prices will the firm supply zero output?

The firm will supply zero output when price is below average variable cost ( $P < AVC$ );  
 The minimum of  $AVC = 14$  when  $q = 4$ . Thus, we know that output will be zero if  $P < 14$ .

c. Identify the firm's supply curve on your graph.

The supply curve is portion of the marginal cost curve above the AVC curve.

d. At what price would the firm supply exactly 6 units of output?

When price = \$42,  $q = 6$ .

Since we know they will produce where  $P = MC$ ,

$$P = MC(q) = MC(6)$$

$$MC(6) = 3 \cdot 6^2 - 16 \cdot 6 + 30 = 42.$$

**2. (4 points) A local business owner wonders if she can estimate her long run average costs by examining her short run costs. What would you tell her?**

First, recall that the definition of long run is time it takes for fixed costs, which means capital, to become variable. Average cost in the short run is calculated for a given, constant level of capital. In the long run, a firm's manager chooses the optimal level of capital as well as labor. This can lead to totally different conclusions regarding costs than in the short run.

Figure 8.13 on page 293 illustrates this perfectly: average cost can be rising in the short run due to diminishing MPL but falling in the long run due to economies of scale. In other words, the trend in short run average cost is the opposite of the long run trend, making short run costs a terrible predictor of long run costs in this case.

Why is this case? In the short run, increasing output is achieved by increasing labor. The marginal product of labor is likely diminishing, and thus marginal cost ( $w/MPL$ ) is rising and average cost is also probably rising. Think of a crowded bakery with more and more bakers fighting over an oven. In the long-run, the firm expands the size of the bakery, purchasing more ovens to complement the additional bakers. Average costs fall, perhaps because of bulk pricing of wheat or even ovens, or perhaps due to specialization among bakers (Cookie bakers, cake bakers, bread bakers etc...).

Regardless of the story, economies of scale are often present, while in the short run, the fixed nature of capital and the diminishing marginal product of labor can cause average cost to actually be rising. As such, short run average cost is likely a bad indicator of long run average cost. A better approach might be to consider reasons why her firm would face either economies or diseconomies of scale if output were changed (supplier X has offered a price reduction of Y if I increase purchases by Z, etc...).

**3. (4 points) If a firm's production experiences diminishing marginal product of labor, will its marginal cost be downward sloping?**

No, if we assume the wage rate is not a function of output (a result of uncompetitive factor markets) and that the firm is operating in the short run. Short run marginal cost is defined as  $w/MPL$  (assuming labor is the variable input). For a marginal cost curve to be downward sloping, the expression  $w/MPL$  must decrease. So, a diminishing MPL will only translate into increasing marginal cost if the wage rate is also a constant.

If, on the other hand, wages depend on the number of workers employed by the firm (perhaps labor is scarce). In this case, as hiring more workers to increase output would raise wages, the

marginal cost curve would be upward sloping.

If the firm is operating in the long run, even if input prices do not change as  $q$  changes a diminishing MPL does not guarantee an upward-sloping MC, as long run MC is determined by what occurs to costs as firms alter scale, not what happens to costs as they change a variable input holding others fixed.

**4. (4 points) Why might a firm stay open even if its total revenue is less than its total costs?**

In the long run, a firm will shut down if  $TR < TC$ , but not necessarily the case in the short run. In the long run, if a firm cannot attain a profit of 0 or greater, they will exit the industry.

However, in the short run, certain costs are fixed (often capital investments) and certain costs are variable (the amount of worker-hours is adjustable). The key is that fixed costs are incurred whether the firm shuts down or not. If the firm shuts down, the total cost = the fixed cost = the loss incurred by the firm, as no revenue was generated. However, total revenue exceeds variable cost ( $TR > TVC$ ), then by staying open, they will cover part of the fixed costs ( $TR - TVC =$  portion of TFC covered). If  $TR < TVC$ , then each unit of output sold increase the loss, and the loss minimizing strategy is to shut down.

To summarize:

In the long run, if  $TR < TC$ , the firm will shut down and exit the industry.

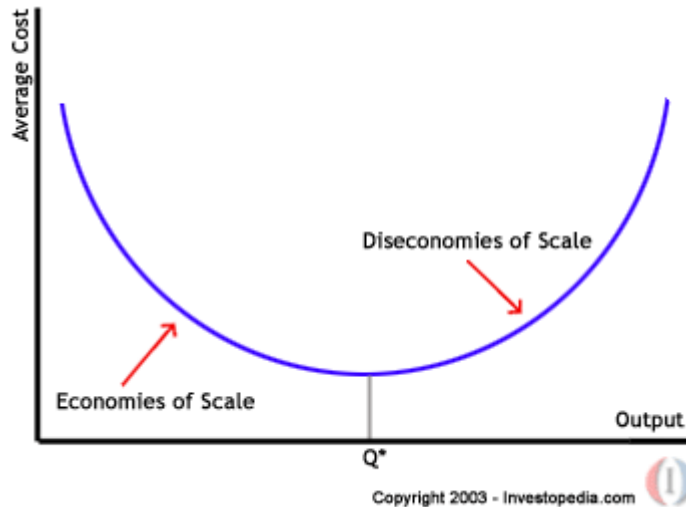
In the short run, if  $TR < TVC$ , the firm will shut down but remain in business.

In all other cases, the firm will remain in business.

**5. (6 points) What does the following quote imply about the cost structure of these firms? “Although the car industry has undergone much consolidation in the past decade, most of it has been disastrous. Two unprofitable firms, Jaguar and Saab, were swallowed up by Ford and GM respectively only to cost billions in further losses” (The London Economist, July 8, 2006, as quoted in James Brock, The Structure of American Industry 2009).**

From the quote, we knew that after Jaguar and Saab’s acquisition by Ford and GM respectively, they still suffered losses. The acquirers tried to grow output in the long run with economics of scale, as long run profit maximization requires that firms produce at the minimum of long run average cost. However, the acquisition only caused more losses.

The graph below shows long run average cost curve with economies of scale:



Thus, the cost structure for the two acquirers has exceeded the optimum  $Q^*$  where there are economies of scale.

**6. (8 points) A perfectly competitive firm that produces roofing nails has production function given by  $q = L^{1/4} K^{3/4}$ , where  $q$  is boxes of nails. The wage paid per worker-hour ( $w$  paid per unit  $L$ ) is \$10, and the cost per unit of capital per hour ( $r$  paid per unit  $K$ ) is \$20. The price per box of nails is  $p = \$12$ , and the firm's current share of total output is 2500 boxes.**

**a. What are the firm's profit maximizing levels of capital and labor? Show all your work and explain how you obtained each step. Use the Lagrangian optimization technique.**

$$\Phi = wL - rK - \lambda[K^{3/4}L^{1/4} - q]$$

Differentiate with respect to  $L$ ,  $K$ , and  $\lambda$ :

$$1) \frac{d\Phi}{dL} = w - \lambda[1/4K^{3/4}L^{-3/4}] = 0$$

$$2) \frac{d\Phi}{dK} = r - \lambda[3/4K^{-1/4}L^{1/4}] = 0$$

$$3) \frac{d\Phi}{d\lambda} = K^{3/4}L^{1/4} - q = 0$$

Solve 1) for  $w$  and 2) for  $r$

$$K = 3L / 2$$

Plug in  $q = 2500$ , so  $L = 1844$ ,  $K = 2767$

**b. What if the firm instead has a production function given by  $q = L + 3K$  (but all else remains as specified above)? What would be its profit maximizing levels of labor and capital? Explain.**

$$MPK = 3, MPL = 1$$

$MPK/r (=3/20)$  is always larger than  $MPL/w (=1/10)$  in this case. Thus, it is cost-efficient for the firm to use all  $K$  and no  $L$

$$\text{As } q = 2500, K = 2500/3 = 833, L = 0$$

**c. How much profit is the firm making?**

For part (a),

$$\text{Revenue} = 12 \cdot 2500$$

$$\text{Cost} = wL + rK = 10 \cdot 1844 + 20 \cdot 2767$$

$$\text{Loss} = \$43,780$$

For part (b),

$$\text{Revenue} = 12 \cdot 2500$$

$$\text{Cost} = wL + rK = 833 \cdot 20$$

$$\text{Profit} = \$13,340$$