Econ 431-01: Public Finance
Prof. Sarah West
50 points

1. (5 points) Joanne has a disposable income of $800 per month. She spends her income on movies at the Grandview Theater and on music downloads. The price of a movie is $12, and the price per music download is $2.00.

a. Graph Joanne's monthly budget constraint. Assume that at the original prices, she spends half her income on movies and half on music downloads. Draw an indifference curve representing this consumption. Why did you draw your indifference curve in the way you did?

See me for the graph. The indifference curve is downward sloping, reflecting the assumption that consumers get positive utility from both goods and therefore to stay indifferent, must give up some of one good when they get more of the other. The indifference curve is also convex, reflecting diminishing marginal rates of substitution, which follows from diminishing marginal utility, which we expect to hold except in extreme cases (e.g. perfect substitutes).

The indifference curve is just barely tangent to the budget constraint. At any non-tangency, it is possible to reallocate spending so that the loss in utility per dollar fewer spent on one good is more than compensated by the increase in utility per dollar increase in spending on the other good, or to increase overall spending.

b. Movies go on sale for $8.00 each. Draw Joanne's new budget constraint and an indifference curve that reflects the fact that for Joanne, the income effect of the price change dominates the substitution effect. Label the income and substitution effects on both axes, and explain.

See me for the graph. When the income effect dominates the substitution effect, a reduction in the price of movies results in an increase in the consumption of movies AND an increase in the consumption of music downloads.

2. (5 points) Tim consumes two goods, x and y, whose prices are respectively p_x and p_y. His income is I. Tim's utility function can be represented as:

$$U = x^{1/3} y^{2/3}$$

a. If Tim gains one unit of good x, how much less of good y must he consume to hold his utility constant? Explain.

The answer to this question is given by the marginal rate of substitution between *x* and *y*, which is equal to the ratio of marginal utilities:

MUx/MUy = y/2x

Thus, the specific answer depends on the current level of consumption of *x* and *y*. Tim should consume *y/2x* fewer units of good *y* to offset the utility gain from getting one unit of good *x*. This makes sense—as the starting amount of x rises (and y falls), Tim needs less y to offset the utility gained with a one unit increase in x (as in Tim's bundle y is relatively more scarce and x relatively more plentiful).

b. Use the Lagrangian optimization technique to derive expressions for Tim's demand for each of the two goods in terms of prices and income.

$$\max_{x,y,\lambda} \theta = x^{1/3}y^{2/3} - \lambda\left[P_x x + P_y y - I\right]$$

$$\frac{\partial\theta}{\partial x} = \frac{1}{3}x^{-2/3}y^{2/3} - \lambda P_x = 0 \qquad (1)$$

$$\frac{\partial\theta}{\partial y} = \frac{2}{3}x^{1/3}y^{-1/3} - \lambda P_y = 0 \qquad (2)$$

$$\frac{\partial\theta}{\partial\lambda} = -P_y y - P_x x + I = 0 \qquad (3)$$

Kill the lamdas using (1) and (2):

$$\frac{1}{3}x^{-2/3}y^{2/3}/\frac{2}{3}x^{1/3}y^{-2/3} = \lambda P_x/\lambda P_y$$

$$y/2x = P_x/P_y$$

Cross-multiply:

$$2P_x x = P_y y \qquad (4)$$

Plug (4) into (3):

$$P_x x + 2P_x x = I$$

Solve for x:

$$x = \frac{I}{3P_x} \qquad (5)$$

Use similar steps to solve for y:

$$y = \frac{2I}{3P_y} \qquad (6)$$

c. Show that the log-log form of this utility function yields the same demands.

See me for help.

d. If Tim's utility function is instead given by the following equation, how much of good x and how much of good y will he consume?

$U = \min(4x, 5y)$

Now for Tim, goods x and y are perfect complements. To maximize utility, he should always consume those goods so that $4x = 5y$.

To solve for the amounts of the goods, you would use this equation and the budget constraint (two equations, two unknowns).

3. (9 points) Your professor does research on the demand for gasoline. She asks for help interpreting the following regression results, where g equals gallons of gasoline, P_g is the price of gasoline, I is income, and standard errors are in parentheses:

$$\log g = .25 \; - .56 \log P_g \; + \; 1.8 \log I \qquad \text{adjusted } R^2 = .28 \quad n = 462$$
$$(.02) \quad (.29) \quad (.50)$$

a. Are the estimated coefficients statistically significant? Use t-statistics in your explanation.

The t-statistic for each coefficient is the coefficient divided by its standard error. For the intercept, the t-stat is equal to 12.5, for the coefficient on gas price the t-stat is -1.93, and for the coefficient on income the t-stat is 3.6. The critical t-value with a confidence interval of 5 percent and 462 data points is equal to approximately 1.96, so after taking absolute values of the t-stats, we can see that the constant term and the coefficient on income are statistically significant at the 5 percent level, but the coefficient on the price f gasoline would be significant at the 10 percent level (testing a two-tailed hypothesis).

b. If the price of gasoline decreases by sixty percent and incomes remain the same, by how much will gasoline consumption change? According to these coefficients, is gasoline a necessity or a luxury? Explain.

We can interpret the coefficients from a log-log regression as elasticities. Thus the price elasticity of demand for gasoline is estimated to equal -0.46. We can use this estimate and the equation for elasticity to solve for the percentage decrease in gasoline consumption:

$-0.56 = \%\Delta\text{quantity}/-.60$

Thus the percentage change in quantity is 33.6 (an increase).

The coefficient on the log of income is greater than one, implying that gasoline is a luxury, or, that as income increases by a specific percentage, the quantity of gasoline consumed increases by a greater percentage. Whether a good is an inferior good, normal good, or a luxury has nothing to do with price elasticities.

Incidentally, I made up these regression results—the income elasticity of demand for gasoline is actually quite small (about 0.20), suggesting that a tax on gasoline is regressive (a tax on gasoline would be progressive if gasoline were actually a luxury).

c. How well does this regression explain the variance in gasoline consumption? Can you suggest any ways in which your professor could improve the regression's accuracy?

The adjusted R^2 indicates that the regression explains about 28 percent of the variation in the dependent variable. As R^2 go, this is not too shabby. Still, the fit could be improved by the addition of a number of explanatory variables, including the number and types of vehicles owned by the household (which are endogenous, so one must instrument for them), household characteristics such as number of commuters, number and age composition of children, region or metro area of residence, etc., the availability and price of substitutes (e.g. public transit).

Ideally one would have panel data on households with very fine location identifiers. This way one can use fixed effects to account for any time-invariant household or location-specific determinants of gasoline consumption.

4. (12 points)

a. Find and describe an example of a public good and an externality on Macalester's campus. Choose your externality so that it is not also a public good or a public bad, and explain why your externality is not a public good or public bad.

Answers will vary. Significant discussion to the rivalness and excludability is key,

5. (13 points) Consider an economy with two people, each of whom consumes one private good, x, and one public good, G. Person 1's utility can be represented:

Consider an economy with two people, each of whom consume one private good, x, and one public good, G. Person 1's utility can be represented:

$$U_1 = \alpha(x_1 + G)$$

while person 2's utility can be represented:

$$U_2 = \beta(x_2 + G)$$

The private good and the public good are produced according to the production function:

$F(X, G) = 0$, *where* $X = x_1 + x_2$

The government's social welfare function is:

$W = 2U_1 + U_2$

a. *Set up and solve the government's optimization problem. What condition characterizes the social optimum? Be sure to use the functional forms given above in your derivation of the optimum. Explain.*

First, note the following:

a.

Person 1 $MRS_{x_1,G} = \dfrac{\partial U/\partial x_1}{\partial U/\partial G} = \dfrac{\alpha}{\alpha} = 1$

and

$MRS_{G,x_2} = \partial U/\partial G \,/\, \partial U/\partial x_1 = \alpha/\alpha = 1$

Person 2 $MRS_{x_2,G} = \dfrac{\partial U/\partial x_2}{\partial U/\partial G} = \dfrac{\beta}{\beta} = 1$

and

$MRS_{G,x_2} = \dfrac{\partial U/\partial G}{\partial U/\partial x_2} = \beta/\beta = 1$

The sum of the marginal rates of substitution is 2.

b.

$$\mathcal{L} = \max_{x_1, x_2, G, \lambda} \; 2\alpha(x_1+G) + \beta(x_2+G) - \lambda(F(X,G))$$

$$\frac{\partial \mathcal{L}}{\partial x_1} = 2\alpha - \lambda \frac{\partial F}{\partial X}\frac{\partial X}{\partial x_1} = 0 \qquad \frac{\partial X}{\partial x_1} = 1$$

$$\frac{\partial \mathcal{L}}{\partial x_2} = \beta - \lambda \frac{\partial F}{\partial X}\frac{\partial X}{\partial x_2} = 0 \qquad \frac{\partial X}{\partial x_2} = 1$$

$$\frac{\partial \mathcal{L}}{\partial G} = 2\alpha + \beta - \lambda \frac{\partial F}{\partial G} = 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = -F(X,G) = 0$$

$$\Rightarrow 2\alpha = \lambda \frac{\partial F}{\partial X} \qquad \beta = \lambda \frac{\partial F}{\partial X}$$

$$2\alpha + \beta = 2\lambda \frac{\partial F}{\partial X}$$

$$\Rightarrow 2\alpha + \beta = \lambda \frac{\partial F}{\partial G}$$

$$2\lambda\, \partial F/\partial X = \lambda \frac{\partial F}{\partial G} \Rightarrow$$

$$2 = (\partial F/\partial G)/(\partial F/\partial X) = MRT$$

From part (a) $MRS_{G,x_1} + MRS_{G,x_2} = 2$

We showed that the MRT is equal to the sum of the MRS. This is finding the Samuelson solution. If you had a functional form for F(X, G), you could solve for the numerical values of the variables.

b. How does the condition for which you solved in part (a) differ from the optimum condition in the case of a market with only private goods? Why do these conditions differ?

In the market for a private good, the marginal rate of transformation (MRT) equals each individual's marginal rate of substitution. In the case of the two-person world in the example above, $MRT = MRS_1 = MRS_2$. In the market for a private good, people maximize utility by choosing the quantity they consume based on the good's price, which results in a ratio of marginal utilities that is equal for everyone and equal to the MRT, but quantities differing across consumers. In the case of a public good, everyone consumes the same quantity (indeed, they consume the very same *thing*), but the price they are willing to pay will vary. As a result, their ratios of marginal utility will not necessarily be equal at the optimal quantity. The optimal quantity, instead, will be where the sum of all of their marginal rates of substitution equals the MRT of providing the good.

c. What does your solution in (a) imply for policymakers? How should policymakers decide how much of a public good to provide? What problems will policymakers likely have in determining how much to provide?

One important implication of the result in (a) is that policymakers need to know the sum of all members of society's marginal rates of substitution to find the optimal provision level of a public good. Obviously, this is no easy task, since reaching all members of society is impossible in itself, let alone the challenge of acquiring their MRS through their utility functions. While the optimization condition is theoretically essential for optimal provision of the public good, its infeasibility requires that we find a way to reasonably approximate the sum of the marginal rates of substitution.

If each person were changed their individual willingness to pay for the public good (their "Lindahl price") then there would be an efficient amount of the public good provided. However, since the good is a public one, enjoyed by all regardless of payment, individuals will always have the incentive to free-ride, to understate their willingness to pay. Such understatement would lead to under-provision of the public good.

The Appendix to Chapter 4 of your Rosen textbook discusses one possible "preference revelation mechanism," a tax scheme that a government might use to encourage households to reveal their true willingness to pay for the public good. Rosen's example, based on Groves and Loeb (1975), induces revelation of WTP only if households understand the complex taxation scheme therein and can compute their own WTP for a public good for all possible levels of the public good. In addition, gathering information on WTP for all households would likely be prohibitively costly.

6. (6 points) Rosen and Gayer, page 107, problem 11.

Private Marginal Benefit = 10 - X

Private Marginal Cost = $5

External Cost = $2

Without government intervention, PMB = PMC; X = 5 units.

Social efficiency implies PMB = Social Marginal Costs = $5 + $2 = $7; X = 3 units.

Gain to society is the area of the triangle whose base is the distance between the efficient and actual output levels, and whose height is the difference between private and social marginal cost. Hence, the efficiency gain is ½ (5 - 3)(7 - 5) = 2.

A Pigouvian tax adds to the private marginal cost the amount of the external cost at the socially optimal level of production. Here a simple tax of $2 per unit will lead to efficient production. This tax would raise ($2) (3 units) = $6 in revenue.