

Econ 431-01: Public Finance
Prof. Sarah West
Homework 3
50 points

1. (25 points) Income Redistribution:

State whether each of the following is true, false, or uncertain. Then explain each of your answers, using equations when appropriate.

a. A risk-neutral individual will prefer to live in a society governed using a utilitarian social welfare function rather than one which uses a Rawlsian social welfare function.

Uncertain. One can interpret Rawls as saying that a Rawlsian framework with Principle 1 (which guarantees a set of basic rights) and a maximin social welfare function will be preferred to utilitarianism without Principle 1. If the playing field is leveled and both approaches guarantee basic rights, then preferences toward risk begin to critically affect the decision between the two approaches. It is the case that a risk-neutral person will be indifferent between obtaining the average utility in society with certainty and a gamble whose expected outcome is equal to the average utility. That person stands to gain as much from an outcome a standard deviation above the average outcome as she stands to lose from an outcome that is a standard deviation below the mean. If this person is in the original position, she will likely be willing to roll the dice, willing to accept a social welfare function that maximizes average utility, which happens with an additive utilitarian social welfare function.

This contrasts with an individual who is risk averse. That person will lose more from the low outcome than she can gain from the high outcome. She will be more likely to prefer the Rawlsian social welfare function, as it protects individuals from the potentially large disutility of the downside of gambles.

b. A society governed using a Rawlsian social welfare function will redistribute to attain a perfectly equal income distribution.

Uncertain. This statement is true if utility functions are identical and functions only of individual income. See the Pete and Karine question below, with the $\min(U_K, U_P)$ social welfare function. The statement is false if utility functions are not identical, that is, if a specific level of income grants greater utility to one person than to another. What is required to maximize this function is that utilities be equal.

If you took the approach as in the paragraph above, for full credit, you should provide an example of each—one with identical utility functions, and one without.

The Rawlsian framework does not completely abstract from the costs of redistribution—if the costs of redistribution are such that redistributing income ends up actually reducing the income of the worst-off person (because they are laid off, for example), then even if utility is a function only of income, redistribution will not be optimal.

c. A redistribution that makes everyone worse off except for the worst-off (who becomes better off) satisfies Rawlsian goals.

Uncertain. This statement is true if the redistribution does not violate Rawls' first principle.

d. Maximizing an additive utilitarian social welfare function is equivalent to maximizing average utility.

True. Average utility is simply the sum of utilities (the additive utilitarian social welfare function) divided by n . The optimum is invariant to such a transformation. For full credit, you should show how the n drops out of the general optimization problem, or given a specific example.

e. In Thurow's model, as long as some individuals prefer an income distribution that is more equal than the current one, income should be redistributed to attain greater equality.

Uncertain. Only if the increase in social welfare due to the redistribution offsets the loss in utility due to reductions in income for those from whom income is taken and the disutility from redistribution for those who dislike redistribution (apart from the effect of the redistribution on their individual incomes) should redistribution take place. That is, redistribution should take place as long as the sum of the marginal rates of substitution between private income and the public good is greater than the marginal rate of transformation between transfer payments and the income distribution (which is the same as saying as long as the marginal benefit of redistribution exceeds the marginal cost, where the marginal cost is the transfer payments to individuals who dislike redistribution). Only if redistribution is Pareto improving, making all individuals at least as well off as they were before the redistribution, should it take place.

2. (25 points) Income Redistribution

An economy consists of two individuals, Pete and Karine, whose utility levels are given by U_P and U_K , and incomes are I_P and I_K respectively. Pete's and Karine's utilities are given by:

$$U_P = 600 I_P - I_P^2$$

$$U_K = 600 I_K - I_K^2$$

Pete's income plus Karine's income = 500.

Consider two social welfare functions:

$$(1) W = U_P + U_K$$

$$(2) W = \min [U_P, U_K]$$

Show (mathematically) that given the above specification of utility functions, maximization of welfare function (1) yields the same results as maximization of welfare function (2).

You all correctly implemented the method for maximizing the first social welfare function, obtaining the optimum when $I_K = I_P = 250$.

To maximize the second social welfare function, note that maximizing the minimum of two utilities is equivalent to maximizing the sum of utilities subject to two constraints, where the first constraint is the income constraint ($I_P + I_K = 500$) and the second constraint is $U_P = U_K$. You can find that the two incomes must be equal by setting the two utility functions equal to each other—only when $I_P = I_K$ are the utilities equivalent. Then it's a matter of splitting the total income of 500 equally between the two. But of course, doing such an optimization problem entails imposing the critical part of the solution ($U_P = U_K$) up front, which is ridiculous. So the best way to do this problem is to give an intuitive explanation.

Intuitively, we can see this because if $U_K < U_P$, we could increase society's total welfare by redistributing income toward Karine to increase U_K (the minimum utility in society). If we give

Karine too much of Pete's income, $U_K > U_P$, and then we could increase society's total welfare by redistributing income toward Pete to increase U_P . To reach a point where the minimum level of utility is maximized, $U_K = U_P$ (assuming, as we've been given, that their utility functions are identical and functions only of income).