CORRECTIONS TO *A RADICAL APPROACH TO REAL ANALYSIS*, 2nd EDITION

page 11: paragraph 4, line 1: “Archimedes” should be “Archimedes’ ”

page 14, Exercise 2.1.1, in part b, the vertices should be at \((a, 1-a^2), (a+\delta, 1-(a+\delta)^2), (a+2\delta, 1-(a+2\delta)^2)\)

page 15: Exercise 2.1.2, last line, coordinates of first point should be \((k2^{-n}, 1-k2^{-2n})\).

page 15: Exercise 2.1.5, line (2.5), the second term should be \(1/2^k\) rather than \(1/(2^k n)\)

page 16: Exercise 2.1.10.a, last line, “are all with” should read ”are all within”

page 17: Exercise 2.1.10 (d) Insert the following sentence immediately before the sentence that begins “Explore the decimal values . . .”: “Given positive integers \(r\) and \(s\), consider the rearrangement of the harmonic series that takes the first \(r\) positive terms, then the first \(s\) negative terms, then continues to alternate \(r\) positive terms with \(s\) negative terms.”

page 25: Equation (2.21), in the second line, the last term should be \(t^6\), not \(t^4\)

page 27: Exercise 2.3.11. This technique only works for \(x \geq 4\). For \(x = 2\) or \(3\), find the square root of \(1/2 = 1 - 1/2\) or \(3/4 = 1 - 1/4\), respectively, then multiply your answer by 2.

page 28: Exercise 2.3.12. \((1+x)^2\) should be \((1+x)^a\).

page 36: Exercise 2.4.9, “greek letter” should be “Greek letter”

page 40: line 3, “Jean Bernoulli” should be “Johann Bernoulli”

page 44: Theorem 2.1. Opening phrase of the theorem should be: “Given a function \(f\) for which the \(n\)th derivative, \(f^{(n)}\), is continuous on an open interval that contains \((a,x), \ldots\)”

page 45: Just above the headline *Lagrange and the Binomial Series*, insert “Of course, we do not need Stirling’s Formula to prove that this limit is 0. For example, choose any integer \(N > 2|x|\). Then show that ratio of consecutive terms is less than \(1/2\) for all \(n > N\), which implies the sequence (for \(n > N\) is bounded above by a constant times \((1/2)^n\) and hence approaches 0.”

page 47: line 1: “\(|x| < 1\)” should be “\(0 < x < 1\)”,

Date: December 20, 2017.
line 3: “If \(|x|\) is larger than 1” should be “If \(x\) is larger than 1”.

page 50: Exercise 2.5.20, change “graph \(x^{-7}\) times each” to just “graph each”

page 52: left-side of last displayed equation should be \(1 - x^2 + x^3 - x^5 + x^6 - x^8 + \cdots\)

page 54: The definition of \(C^p\) and analytic functions ignores a very real distinction between \(C^\infty\) functions and analytic functions. A function \(f\) that is an analytic function at \(x_0\) must be \(C^\infty\) on an open interval containing \(x_0\), but more than that, there must be an open interval containing \(x_0\) in which the power series at \(x_0\) converges to \(f\). The example given on page 55 is precisely an example of a function that is \(C^\infty\) for all \(x\) but is not analytic at \(x = 0\).

page 56: Exercises 2.6.4 and 2.6.5. Delete the adjective “analytic.”

page 58: Figure 3.1, the label at the right endpoint of the interval should be \(x\)

page 59: line 2 should end with “… what do we mean by the”

page 73: In the definition of the intermediate value property, immediately below, and in Figure 3.6: It is confusing to use \(x_1\) and \(x_2\) since \(x_1\) and \(x_2\) occur earlier in the page as points where Cauchy cuts the interval. Substitute \(\alpha\) for \(x_1\) and \(\beta\) for \(x_2\).

page 81: fifth line from bottom, “of \([a,b]\)” should be “on \([a,b]\)”

Page 84: line 16, first term in the sequence should be \(2/\pi\) rather than \(1/\pi\).

Pages 86–7: starting at the second line above equation (3.43), the fact that \(f(x_k)\) and \(f(y_k)\) can be forced as close together as we wish by taking \(k\) sufficiently large relies on uniform continuity, which has not yet been established for continuous functions on closed bounded intervals. To avoid the need to use uniform continuity, replace the text starting at this point and continuing to the end of the proof with the following:

and \(f(x_{k+1}), f(y_{k+1})\) lie on opposite sides of \(A\).

Our sequences \(x_1 \leq x_2 \leq \cdots\) and \(y_1 \geq y_2 \geq \cdots\) satisfy the conditions of the nested interval principle and so there is a number \(c\) that lies in all of these intervals. Again, by the Archimedean definition of limit, we see that

\[
\lim_{k \to \infty} x_k = \lim_{k \to \infty} y_k = c.
\]

Since \(f\) is continuous at \(x = c\), we know that

\[
\lim_{k \to \infty} f(x_k) = \lim_{k \to \infty} f(y_k) = f(c).
\]

Since \(a\) lies between \(f(x_k)\) and \(f(y_k)\) and each of these sequences has the common limit of \(f(c)\), \(A\) must equal \(f(c)\).

page 103, Exercise 3.4.6.f. By “decimal fraction” I mean a number in decimal form
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page 103, Exercise 3.4.12, “increasing” should be “strictly increasing”

page 106: equation (3.52) third term of the series expansion for $F(x)$ should be $f'(a)(x-a)$ rather than $f(a)(x-a)$.

page 107: Theorem 3.11. The hypothesis should be that there is a neighborhood of $x = a$ in which all derivatives of $f$ exist rather than just that all derivatives of $f$ exist at $x = a$.

page 109: In “Definition: infinite limit and limit at infinity,” line 4 should read sufficiently close to $a$ (but not equal to $a$). That is to say, there is a $\delta > 0$ so that $0 < |x-a| < \delta$ implies that

page 112, exercise 3.5.3, second displayed inequality, condition should read “if $0 < \alpha < 1$”

page 113, Exercise 3.5.8, last expression in displayed equation should be

$$\lim_{x \to 0} \frac{2x \sin(1/x) - \cos(1/x)}{1},$$

page 122: first paragraph following Theorem 4.1: ... I want to emphasize what it [omit is] does

page 127, 3rd line before exercises, change “The summands alternate” to “The signs of the summands alternate”

page 131: Corollary 4.8 (The Limit Ratio Test). Add: If the limit does not exist, then this test is inconclusive.

page 132: Corollary 4.10 (The Limit Root Test). Add: If the limit does not exist, then this test is inconclusive.

page 158: Exercise 4.3.27, “those values of $k$” should be “those values of $m$”

page 161: last line before Abel’s Lemma, the month of Abel’s death should be April, not January.

page 172: equation (5.5), in the limit after the equal sign, $\lim_{x \to 0}$ should be $\lim_{y \to 0}$

page 177, line 3 following ”Rearrangement with Conditional Convergence,” (5.13) should be (5.11)

page 193, Figure 5.7. There is no graph of $y = x$. Delete the phrase “, with graph of $y = x$ included”

page 195, Theorem 5.7, part 1 should read $F = f_1 + f_2 + f_3 + \cdots$ converges uniformly over any bounded subinterval of $I$
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page 201: Exercise 5.3.3 should read: Prove that if \( \sum g_k \) converges uniformly over the bounded interval \( I \) and if \( f_k(x) = (x - a)g_k(x) \), then \( \sum f_k \) converges uniformly over \( I \).

page 202: Exercise 5.3.11. The reference should be to exercise 5.3.10, not 6.3.7.

page 212: Exercise 5.4.1.b., summand should be \( \frac{n^2}{\sqrt{n!}}(x^n + x^{-n}) \), missing factorial in the denominator.

page 229: The proof of Lemma 6.3 contains an error. The fact that \( x \) is the upper limit of the \( x_n \) does not guarantee that \( |x - x_n| \) can be made arbitrarily small for sufficiently large \( n \). To see a corrected proof, go to www.macalester.edu/aratra/corrections/lemma6-3.pdf.

page 231: first line, “We have to careful.” should be “We have to be careful.”

page 232: First displayed equation after (6.32) should be \( |F(x + 0) - f(x + 2a)| \), missing closing parenthesis.

page 233: Definition of \( g \) in first displayed equation, top line should be \( |F(x + 0) - F(x + 2u)|, 0 < u \leq a, \)

page 256, Last line of exercise 6.3.9, the functions \( f \) should be \( h \).

page 240: second to last displayed inequality: In the first summation, the upper limit of summation should be \( r \) rather than \( n \).


page 274: Equation (A.8), the numerator of the fraction to the right of the equality should be \( 4 \cdot 6 \cdot 8 \cdots (2p + 2q - 2) \)

page 279: first display after (A.21): upper limit of integration should be \( k \) [not 1]

page 310: hint for 4.2.5, “(f)” should be “(d)”