The Quadrature of the Parabolic Segment

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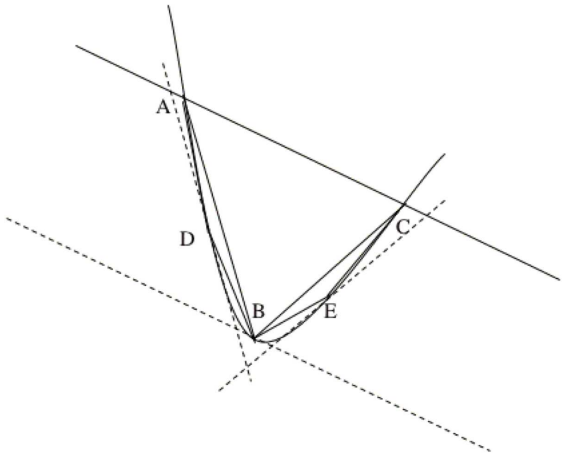
The problem of quadrature is one of finding the square or rectangle whose area is the same as that of a given region. In other words, it amounts to finding the area of the given region.Archimedes considered an arbitrary region bounded by an arc of a parabola, $ADBEC$, and a straight line, AC.



The point B is on the arc of the parabola where the tangent line is parallel to line AC, D is the point where the tangent line is parallel to AB, and E is the point where the tangent line is parallel to BC. Archimedes proves that the areas of triangles ADB and BEC are each 1/8 of the area of triangle ABC. This argument can then be repeated to show that each of the next four triangles are each 1/8 of the area of ADB or BEC, or 1/64 of the area of ABC. At the nth step, we add 2^n triangles, each of which has area equal to $1/8^n$ of triangle ABC.

You can find how Archimedes proved that triangle ADB has 1/8 the area of triangle ABC on pages 54–57 of *Archimedes: What did he do besides cry Eureka?* by Sherman Stein.[1]

[1]Published by MAA, 1999.