HONORS PROJECT PROPOSAL

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My honors project will focus on extremal structures for random walks on trees. In particular, I will attempt prove which graph maximizes the mixing measure $T_{bestmix}$. The access time $H(i, \pi)$ is the expected length of the stopping rule from a node i to the standard distribution π. When studying random walks, we are often interested in how long it takes for various graphs to become sufficiently mixed; in other words for a given graph G we would like to know the value of $H(i, \pi)$ for an appropriate node (or nodes). The node we choose depends on what we take to be our mixing measure. For example, the mixing time, $T_{mix} = \max_i H(i, \pi)$, is the [access/hitting] time given the "worst" initial node. Depending on the circumstance, we may instead want to use the "best" node or the average over all nodes. What are the extremal structures for each of these calculations? That is, for a given number of vertices, what graph maximizes or minimizes the value of each mixing measure?

In their paper, "Exact mixing times for random walks on trees," Meng Wang (Macalester, 2009) and Andrew Beveridge show that out of trees with n vertices, the path P_n maximizes T_{mix} and the star $S_n = K_{1,n-1}$ minimizes T_{mix}. T_{mix} describes the length of the optimal stopping rule given the "worst" initial node (i.e. the node i for which $H(i, \pi)$ is maximized.) A different mixing measure, $T_{bestmix} = \min_i H(i, \pi)$, considers instead the "best" initial node, and describes the length of the optimal stopping rule given that node. The star clearly still minimizes this quantity. It remains an open question, however, which structure maximizes $T_{bestmix}$. Based on calculations of specific cases, the path is surprisingly not the structure that does so for odd n. The conjecture is that the path still maximizes $T_{bestmix}$ for all even n, but that the $(n-1)$-path with a leaf adjacent to one of the center vertices in fact achieves the maximum value of $T_{bestmix}$ for all odd n. The goal of my honors project is to prove this conjecture for the general case.

Over the summer I will carefully read through Meng's honors thesis, as well as Professor Beveridge's "Centers for random walks on trees," Lovasz and Winkler's "Efficient stopping rules for Markov chains," and O. Haggstrom's *Finite Markov Chains and Algorithmic Applications*. I will calculate some examples for the key results in the papers, as well as begin preliminary investigations on the open problem. In the fall I will register for a two credit independent study with Professor Beveridge. We will meet twice a week to discuss results and set new deadlines. We plan to obtain the majority of our results by the end of the fall semester. The January term will serve as a period in which to finish up calculations and conclusions, and to begin putting the paper together. For the most part, we will devote the spring semester to the writing of the paper itself in order to have a finished product by the deadline in March.

References

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[2] Beveridge, A., Wang, M.: Exact mixing times for random walks on trees, submitted.

[3] Haggstrom, O.: Finite Markov Chains and Algorithmic Applications. Cambridge University Press, Cambridge UK, (2002).

[4] Lovasz, L., Winkler, P.: Efficient stopping rules for Markov chians. Proceedings of the 27th ACM Synposium on the Theory of Computing, 76-82 (1995).