

# Representation Theory of the Rook Brauer Algebra

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## 1 Project Description

The Brauer algebra is represented on diagrams on two rows of  $n$  dots. The diagrams differ from the representation of the permutation group  $S_n$  in that pairs of dots in the same row can be connected. Therefore, the basis of the Brauer algebra represented by a diagram with  $2n$  dots has a basis that consists of all possible matchings of the  $2n$  dots. An algebra consisting of  $2n$  elements has a dimension of  $(2n)!/2^n n!$ . The Brauer algebra has a strong connection to the orthogonal group  $O(r, \mathbb{C})$ , which consists of all  $r \times r$  orthogonal matrices with elements in  $\mathbb{C}$ .

I intend to study the "rook" Brauer algebra, which differs from the Brauer algebra in the way that diagrams are constructed. The term "rook" comes from the rook algebra, which will be a sub-algebra of the rook Brauer algebra. The dimension of the rook algebra is all possible ways to place non-attacking rooks on a chess board. In the Brauer algebra, each diagram must be complete. Every vertex must be matched to another vertex. However, in the rook Brauer algebra there can be unmatched vertexes in the diagram. For example, in a diagram consisting of two rows of two dots, the upper left and lower left dots could be connected while the upper right and lower right dots remain unmatched. I propose to explore this particular version of the Brauer algebra in order to learn more about its properties. One of my goals is to find the irreducible representations of the "rook" Brauer algebra. I will also try to determine its relationship to the group  $O(r, \mathbb{C})$ . Since the Brauer algebra has such a strong connection to the orthogonal group, it is likely the "rook" Brauer algebra will as well. Finally, I will find the conjugation representation of the "rook" Brauer algebra and generalize it.

## 2 Timeline

*Summer:* Professor Tom Halverson and I intend to communicate over the summer about the project and start doing calculations. I will conduct preliminary research and familiarize myself with both the Brauer algebra and the "rook" Brauer algebra.

*Fall Semester:* I will hopefully begin looking into my research goals during the summer and will start working in-depth when the Fall semester begins. I will also be taking MATH 476 (Topics in Modern Algebra) and will be working on the research as my project for the class.

*Winter Break:* I intend to begin writing the paper for the project by the beginning winter break so that I can feel confident about finishing the paper by the middle of Spring semester.

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### 3 Possible Sources

#### References

- [1] Barcelo, H. & Ram A. (1999). Combinatorial Representation Theory. *New Perspectives in Geometric Combinatorics*, 38, 23 - 90.
- [2] Ram, A. (1995). Characters of Brauer's Centralizer Algebras. *Pacific Journal of Mathematics*, 169(1), 173 - 199
- [3] Goodman, R. & Wallach, N. R. *Symmetry, Representations, and Invariants*. Springer, Dordrecht (2009).