Math 477  Homework # 4  due February 21

(1) What are the problems with Fourier series and why are they problems?
(2) Give proofs of the three theorems on page 10, being certain to clearly state all assumptions.
(3) Explain why it is that if for every interval \([a, b] \subseteq [-\pi, \pi]\), there exists \(r\) and \(s\) such that \(a < r < s < b\) and \(f\) is continuous on \([r, s]\), then the points of discontinuity of \(f\) in \([-\pi, \pi]\) are nowhere dense.
(4) Prove that if \(f\) is monotonic on \([a, b]\), then it is Riemann-integrable over this interval.
(5) What is the difference between a Fourier series and a trigonometric series?
(6) What is meant by uniform convergence in general and how does this differ from uniform convergence?
(7) Give examples of sets of type 0, 1, and 2.
(8) Prove that a set of type \(n\) must be nowhere dense.
(9) Summarize du Bois-Reymond’s accomplishment (described on page 26) and explain why it might suggest that there are many functions with trigonometric series representations that are not given by their Fourier series.