Math 477  Homework # 6  due March 7

Sections 3.1 and 3.2.

(1) Prove that Volterra’s set (page 56) is nowhere dense and has positive outer measure.

(2) Explain what Harnack thought he had proven as a generalization of Theorem III.

(3) Explain why Harnack thought he had proven the existence of an interval which is the union of non-overlapping intervals and whose length is strictly greater than the sum of the lengths of the non-overlapping intervals. Where is the flaw in his reasoning?

(4) What is the difference between a property that holds “in general” and one that is true “almost everywhere”?

(5) Consider the following three characteristics of a subset of $[0,1]$: it either is or is not of first species, it either is or is not nowhere dense, it either is or is not of measure 0. There are eight combinations of these characteristics. For each combination, either explain why there are no sets with this combination or give an example of a set that has this combination of characteristics.

(6) Prove that if $f$ is Riemann integrable over $[a,b]$ and $P$ is any partition of $[a,b]$, then each interval in $P$ contains a point $t_i$ at which $f$ is continuous.

(7) Prove that Weierstrass’s integral is not always additive: find functions $f$ and $g$ for which

$$(W) \int_a^b (f + g) \neq (W) \int_a^b f + (W) \int_a^b g.$$