Math 477  Homework # 9  due April 4

(1) Prove that the characteristic function of the rationals is not integrable under Peano’s definition of the integral.

(2) Let
\[ f(x, y) = \begin{cases} 
1, & \text{if } x \text{ rational}, \\
2y, & \text{if } x \text{ irrational}. 
\end{cases} \]

Show that
\[ \int_0^1 \left( \int_0^1 f(x, y) \, dy \right) \, dx = 1, \]
while
\[ \int_0^1 \left( \int_0^1 f(x, y) \, dx \right) \, dy \]
does not exist.

(3) Explain Jordan’s example of a set \( E \) for which \( \int_E 1 = 0 \) but the iterated integral is 1.