

Your solutions must be uploaded onto Moodle by 5pm CST, Friday, December 18.

This exam is worth 100 points. Show your work. Partial credit will be given for partially correct solutions. You may use a calculator, but you also may leave any numerical quantities or answers in unsimplified form. You may use your textbook and notes, but may not talk with anyone about these questions or use any other resources.

1. On the first line of your exam, copy the following statement, followed by your name.

I certify that this represents my work and only my work. NAME:

2. [5 pts.] A table of values for f , g , f' , and g' is given.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	2	2	3	6
2	1	8	1	7
3	5	2	7	9

If $h(x) = g(f(x))$, find $h'(1)$. Show your work.

3. [15 pts.] Below are the differential equations for an S-I-R model

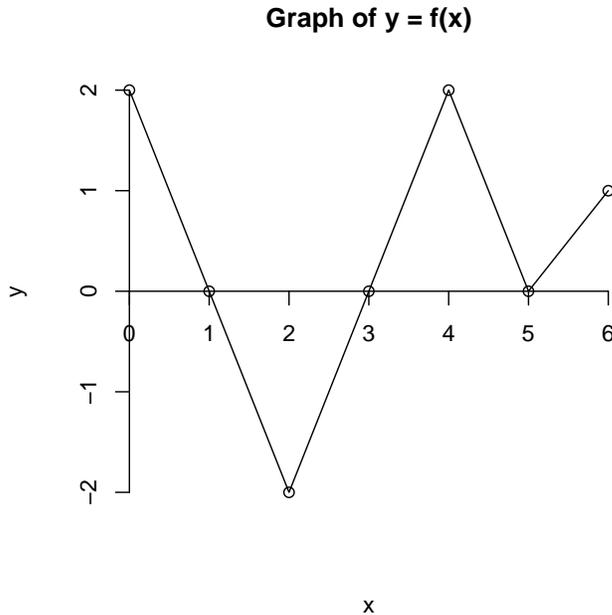
$$\frac{dS}{dt} = \quad - \quad 0.10SI$$

$$\frac{dI}{dt} = 0.10SI - 0.30I,$$

where S and I are measured in millions of people.

- (a) Explain what the constant 0.10 represents in terms of this problem
- (b) Explain what the constant 0.30 represents in terms of this problem
- (c) What is the threshold value for this model?
- (d) Explain in your own words why there is an epidemic if S is greater than the threshold value and why there isn't an epidemic if S is less than the threshold.
- (e) Explain in your own words why immunization can prevent an epidemic even if not everyone gets immunized. If the total population is 10 million people, how many would need to be immunized to avoid an epidemic?

4. [15 pts.] Let $a(z)$ be the area accumulation function, $a(z) = \int_0^z f(x) dx$, where the graph of $y = f(x)$ is shown below.



- (a) Find $a(2)$ and $a'(2)$.
- (b) Find all values of z in the open interval $0 < z < 6$ at which $a(z)$ attains a local maximum. Justify your answer.
- (c) Find the value of z at which $a(z)$ attains the global maximum over the closed interval $0 \leq z \leq 6$. What is the value of $a(z)$ at this point?
5. [15 pts.] Cafe Mac sells lattes for p dollars. In your work study job, you see that the number of lattes sold each day depends on the price according to some unknown function f ,

$$q = f(p) \quad (\text{the number of lattes sold, } q, \text{ is a function of the price } p).$$

The current price of a latte is \$2.50, and you have estimated that

$$f(2.50) = 40 \quad \text{and} \quad f'(2.50) = -20.$$

The revenue $R(p)$ is the *product* of the price p and the quantity sold:

$$R(p) = p \cdot f(p)$$

Determine $R'(2.50)$. Would they increase their revenue if they raise the price to \$2.60? Explain why or why not.

6. [15 pts.] The rate at which a drug leaves the bloodstream and passes to the urine is proportional to the quantity of the drug in the blood at that time.
- Write a differential equation for the quantity, Q , of the drug in the blood after t hours.
 - Suppose that, if an initial dose Q_0 is injected directly into the blood, 40% is left in the blood 2 hours later. Also, suppose that the patient is given 50 mg initially. In terms of a formula that describes Q as a function of t , what is the solution to the equation you gave in (a)?
 - How much of this drug is in the patient's body after 6 hours?
7. [20pts.] You are working on a project to improve outcomes in schools. You have two policy controls: (1) the class size and (2) the salaries you pay to teachers (which affects the incentive that qualified people have to become and remain teachers). In addition to improving the school outcome, you have to deal with legislators who want to spend as little money as possible.

The graph on the next page shows two functions:

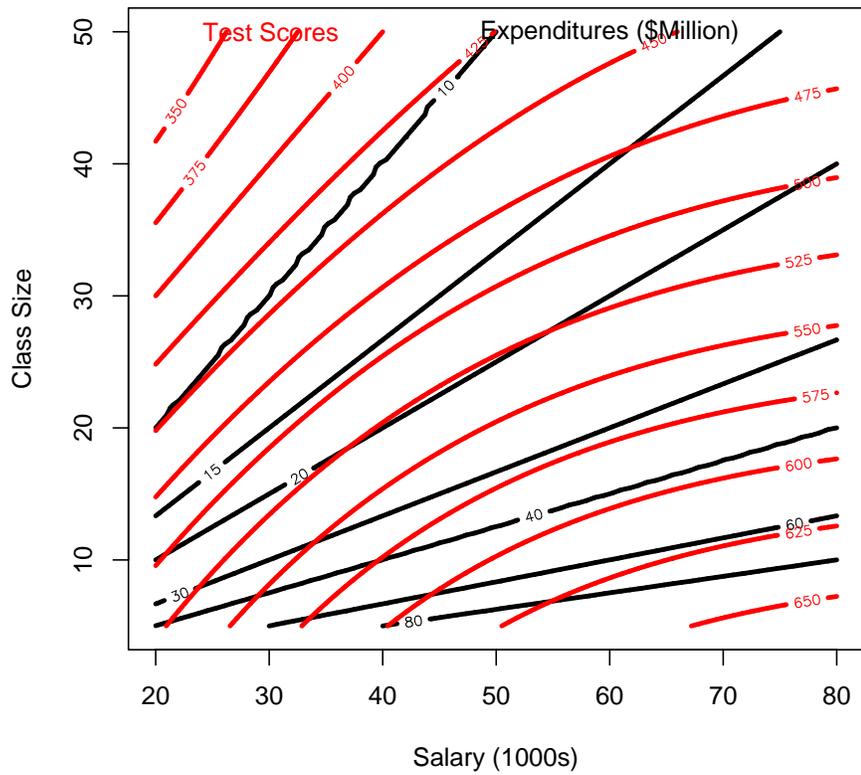
Black The total expenditure for any combination of class size and teacher salary.

Red The average standardized test score in the district.

The expenditure function is straightforward since it's easy to calculate how many teachers the district needs to support any given class size.

The test-score function is more speculative.

- Does this graph of test-score function follow conventional wisdom, namely that test scores should go up as class size gets smaller or as teacher salary goes up? Justify your answer in terms of the features of the graph.
- Using the graph, estimate at the point (Salary = 40, Class Size = 15) the values of the two partial derivatives, $\frac{\partial \text{Expenditure}}{\partial \text{Salary}}$ and $\frac{\partial \text{Expenditure}}{\partial \text{Class Size}}$. You should give numbers for each derivative (though they will have to be approximate). For simplicity, just treat Salary as values like 40, 50, ... without worrying about the \$1000s.
- The legislature is willing to spend \$20 million per year. Sticking to this budget, they propose to set teacher salary at \$30 thousand per year and class size at 15 kids per class. Is there a better arrangement of salary and class size that sticks to the same budget? Explain why or why not and indicate what you think would be the best arrangement.
- A civic action group, "Citizens for Better Education," wants the legislature to increase the school funding level to \$30 million. Another group, "People For the Arts," says the extra \$10 million would be better spent on extra-curricular cultural activities. Both groups say that their initiatives would improve test scores. In order to compare the two claims, you need to calculate the schools' efficiency for improved test scores: dollars per point increase in test score. Using the graph, estimate this ratio when by considering the increase in test scores when funding increases from \$20 million to \$30 million. Give a numerical value and explain how you did the calculation.



8. [15 pts.] What do you consider to be the most important result that you learned in this course? Explain this result at a level that could be read by someone who has studied some calculus but has not been in this course.