

This exam is worth 100 points. Show your work. Partial credit will be given for partially correct solutions. You may not use a calculator, so leave any numerical answers in unsimplified form.

Name: _____

1. Residents of the town of Maple Grove who are connected to the municipal water supply are billed a fixed amount each month plus a charge for each cubic foot of water used. A household using 1000 cubic feet was billed \$40, while one using 1600 cubic feet was billed \$55.

(a) What was the charge per cubic foot?

(b) Write an equation that describes the total cost of a resident's water as a function of cubic feet of water used.

(c) How many cubic feet of water used would lead to a bill of \$100?

2. For each of the following statements, state whether it is true or false, and then give a reason for your answer (no credit for simply answering true or false).

(a) If $y = f(x)$ is an exponential function and if increasing x by 1 increases y by a factor of 5, then increasing x by 2 increases y by a factor of 10.

(b) An exponential function can be decreasing.

3. Let $f(v)$ be the gas consumption (in liters/km) of a car going at velocity v (in km/hr). In other words, $f(v)$ tells you how many liters of gas the car uses to go one kilometer when it is traveling at v km/hr.

(a) What are the units of $f(80)$ and $f'(80)$?

(b) Given that $f(80) = 0.05$, what does this tell you about gas consumption?

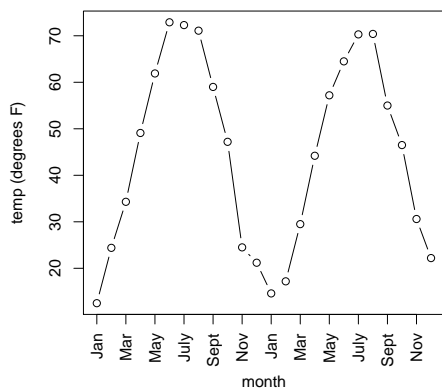
(c) Given that $f'(80) = 0.0005$, what does this tell you about gas consumption?

4. The table and plot below show the average temperatures for two consecutive years, for St. Paul. We want to model the average monthly temperature in Saint Paul using a trigonometric function.

Table 1: Average temperature, St. Paul

month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
temp.(°F)	12.5	24.4	34.3	49.1	61.9	72.9	72.3	71.1	59.0	47.2	24.5	21.2

month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
temp.(°F)	14.6	17.2	29.5	44.2	57.2	64.5	70.3	70.4	55.0	46.5	30.6	22.2



- (a) What is the approximate period (specify units)?
- (b) What is the approximate amplitude of the model (specify units)?
- (c) Let $T(t)$ denote the average temperature at time t . Use the values in parts (a) and (b) to write T as a function of t .

5. A hydroelectric generator captures some of the energy released by falling water. We have the following unit conversions:

- 1 foot · pound is the energy released when 1 pound of water falls 1 foot
- 1 joule is the energy released when 1 newton of water falls 1 meter
- 1 pound = 4.448 newton
- 1 meter = 3.281 feet
- 1 cubic foot of water weights 62.5 pounds.

How much energy in joules is released when 5000 cubic feet of water fall down an 80 foot waterfall? Express your answer as a product, including all relevant units. Do not simplify your answer.

6. The power, P , generated by a windmill depends on three variables:

- The air density, ρ ,
- The length of the windmill blades, L ,
- The windspeed, w .

Create a dimensionally correct model for the windmill power of the form

$$P = k\rho^a L^b w^c$$

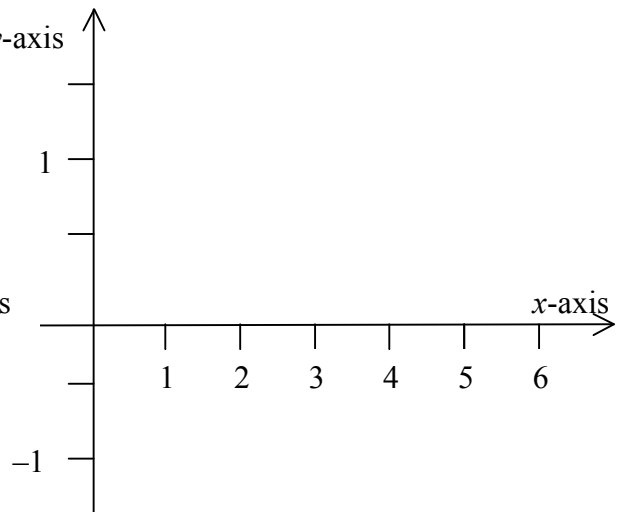
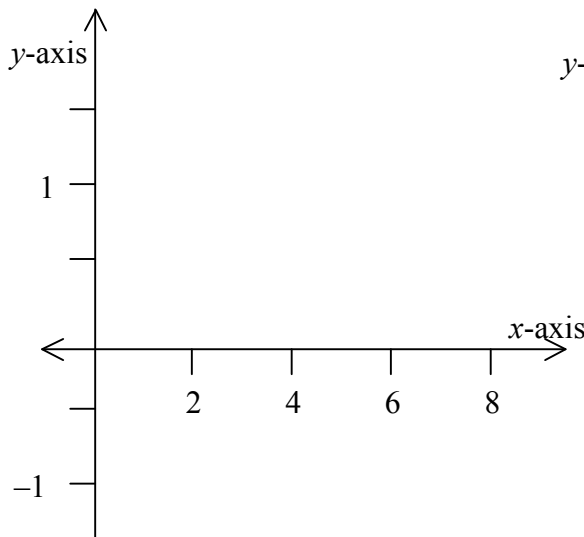
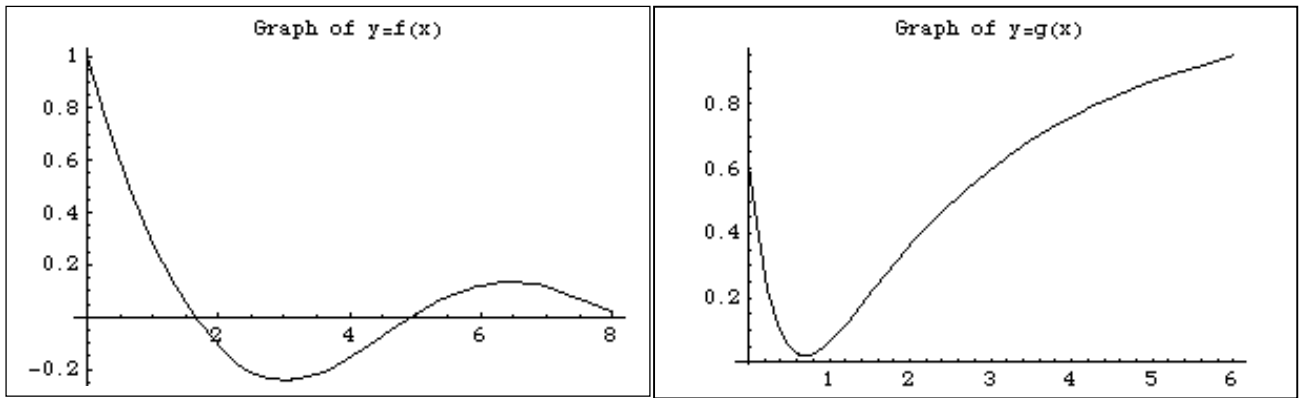
where k is a dimensionless constant. Remember that force is mass times acceleration, work is force times distance, and power is work per unit time.

7. Graphs of the functions f and g are shown below.

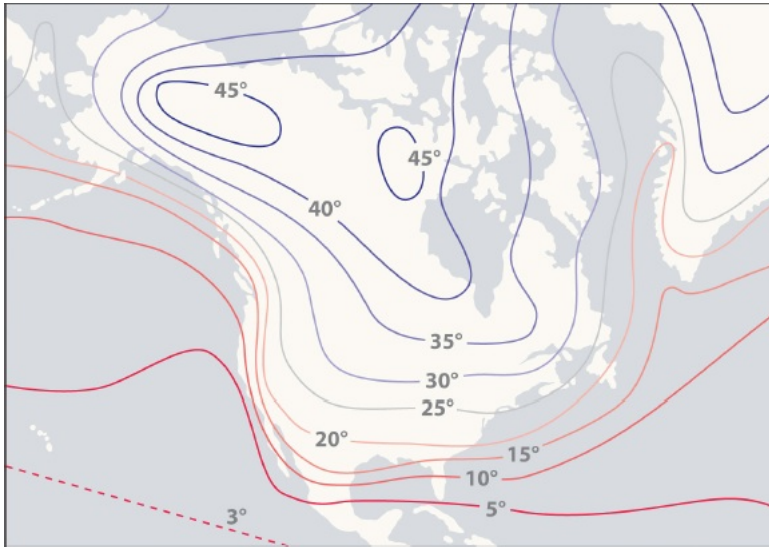
(a) For what values of x is the derivative, f' , positive?

(b) At approximately what values of x does the derivative, g' , have it greatest and least values?

(c) Sketch the graphs of f' and g' .



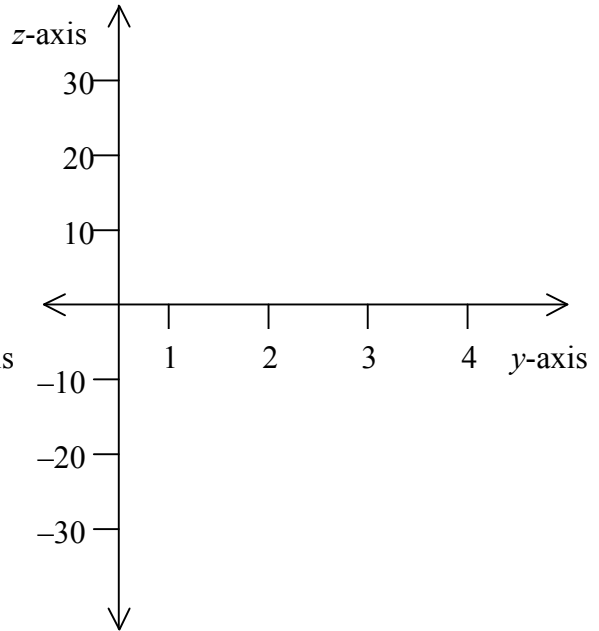
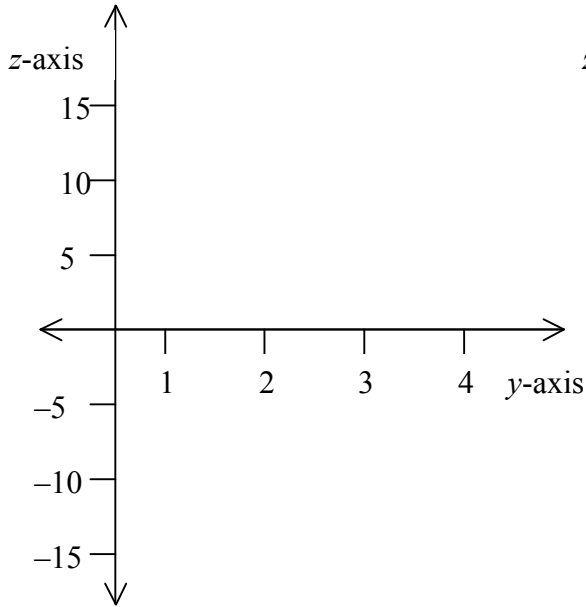
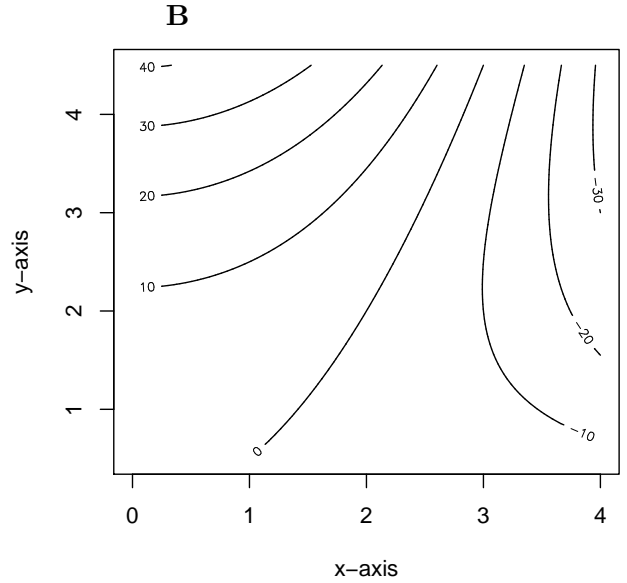
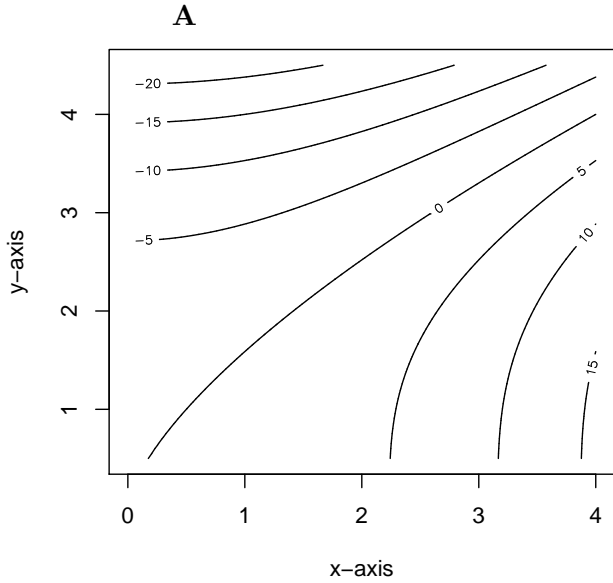
8. The contour diagram shows the average Celsius temperature difference between January and July in North America.



- (a) The contour diagram represents a two-variable function. What are the input variables?
- (b) Where is the temperature difference the greatest?
- (c) What is the effect of land and sea on the difference?
- (d) The contour lines are closest together where North America comes close to the northern Pacific Ocean. What does this tell us?

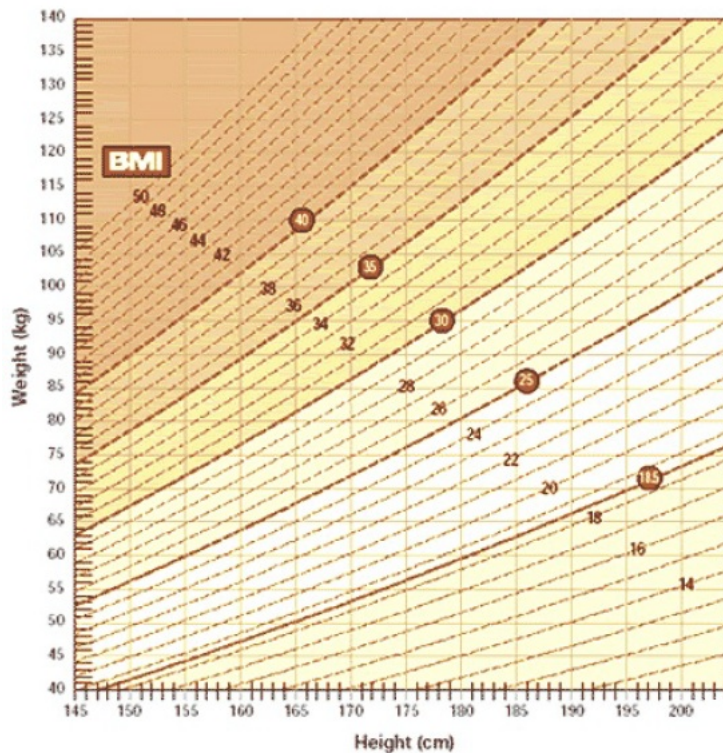
9. Contour plots for two functions, each representing z as a function of x and y , are shown below.

(a) For each contour plot, sketch the graph of the cross-section of z as a function of y when x is held constant at $x = 3$.



(b) Which of these (**A** or **B**) could be the contour plot of the function $F(x, y) = x^2 - y^3/4$? Give a reason for your answer.

10. The figure shows a contour diagram of an adult human Body Mass Index (BMI) (in bmi units) as a function of height, h in cm, and weight, w in kg.



- (a) Is $\frac{\partial BMI}{\partial h}$ positive, negative, or very close to 0?
- (b) Is $\frac{\partial BMI}{\partial w}$ positive, negative, or very close to 0?
- (c) What are the units for $\frac{\partial BMI}{\partial w}$?
- (d) What is the approximate value of $\frac{\partial BMI}{\partial w}$ for a person who is 180 cm tall and weighs 80 kg? Show the work that leads to your answer.