Math 135, Applied Calculus
Sections 01 & 02 (Bressoud)

Second Midterm Exam
13 November, 2015

This exam is worth 100 points. Show your work. Partial credit will be given for partially correct solutions. You may not use a calculator, so leave any numerical quantities or answers in unsimplified form.

Name:____________________________________

1. **[10 pts]** Using the values of f, f', g, and g' in this table,

x	1	2	3	4
$f(x)$	3	2	1	4
$f'(x)$	1	4	2	3
$g(x)$	2	1	4	3
$g'(x)$	4	2	3	1

, find:

(a) $h(4)$ if $h(x) = f(g(x))$

(b) $h'(4)$ if $h(x) = f(g(x))$

(c) $h'(4)$ if $h(x) = g(x)/f(x)$

(d) $h'(4)$ if $h(x) = f(x)\,g(x)$

2. **[10 pts]** Find a linear approximation to $y = xe^{(x^2)}$ that is valid near $x = 3$.

3. **[10 pts]** The voltage, V, in volts, in an electrical outlet is given as a function of time, t, in seconds, by the function $V = 156\cos(120\pi t)$.

   (a) Give an expression for the rate of change of voltage with respect to time.

   (b) Is the rate of change ever zero? Explain.

   (c) What is the maximum value of rate of change? (Note: this asks for the maximum of the rate of change, not the maximum voltage.)

4. **[10 pts]** A yam is put in a hot over, maintained at a constant temperature of 200° C. At time $t = 30$ minutes, the temperature T of the yam is 120° and is increasing at an (instantaneous) rate of 2°/min. Newton's law of cooling (or, in our case, warming) implies that the temperature at time t is given by $T(t) = 200 - ae^{-bt}$. Find a and b.

5. **[12 pts]** Find and classify the critical points of $f(x) = x^3(1-x)^4$ as local maxima, local minima, or neither. Justify your answers.

6. **[15 pts]** State whether each statement is true or false and **give a reason for your answer**.

(a) If $f_x(a,b) = f_y(a,b) = 0$, then (a,b) is a local maximum or a local minimum of f.

(b) The function $f(x,y) = x^2 - y^2$ has a local minimum at $(0,0)$.

(c) Every function has at least one local maximum.

(d) If $f(x,y)$ has a local maximum at (a,b) subject to the constraint $g(x,y) = c$, then $g(a,b) = c$.

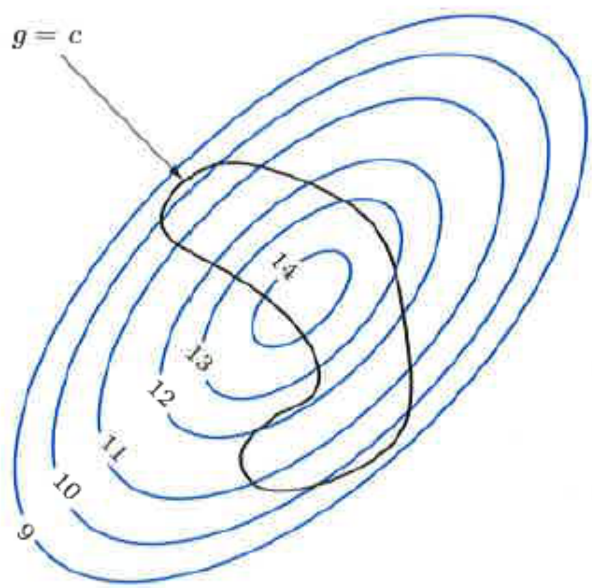
(e) If M is the maximum value of f subject to the constraint $g(x,y) = c$ and if (a,b) satisfies both $f(a,b) = M$ and $g(a,b) = c$, and if $f_x(a,b)/f_y(a,b) = 5$, then $g_x(a,b)/g_y(a,b) = 5$.

7. **[9 pts]** The graph shows contours labeled with values of $f(x,y)$ and a constraint $g(x,y) = c$. Mark the appropriate points at which

   (a) Mark with an a those points where $(f_x, f_y) = \lambda(g_x, g_y)$

   (b) Mark with a b those points where f has a maximum

   (c) Mark with a c those points where f has a local maximum on the constraint $g = c$.



8. **[12 pts]** You have set aside 20 hours to work on two class projects. You want to maximize your grade (measured in points), which depends on how you divide your time between the two projects.

   (a) What is the objective function for this optimization problem and what are its units?

   (b) What is the constraint?

   (c) Suppose you solve the problem by the method of Lagrange multipliers. What are the units for λ?

   (d) What is the practical meaning of the statement $\lambda = 5$?

9. **[12 pts]** A 200-lb weight is attached to a 20-foot rope dangling from the roof of a building. The rope weighs 2 lb/ft. We want to find the work done in lifting the weight to the roof. Remember that as you pull in the rope, there is less of it that has to be lifted.

   (a) What are we accumulating in this problem?

   (b) What product gives us the quantity we are accumulating?

   (c) What sum can we use to underestimate the amount of work, using 100 intervals?

   (d) What is the definite integral that represents the answer to this problem, and what are the units of the answer?