Calculus Reform: What is different this time?

Version of 7/8/2019, submitted to the *Notices of the AMS*

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On May 30–31, 2019, representatives of the departments of mathematics at 24 public and private universities met in Lincoln, Nebraska to share department-wide efforts to improve teaching and learning in their precalculus through single variable calculus sequence. These departments are working either with *Progress through Calculus* (PtC, NSF # 1430540), run by the Mathematical Association of America (MAA), or *Student Engagement in Mathematics through an Institutional Network for Active Learning* (SEMINAL, NSF #'s 1624643, 1624610, 1624628, and 1624639), a project of the Association of Public and Land-Grant Universities (APLU). Representatives reported on course coordination, collection and use of data, variations in course structure, training for graduate teaching assistants, culturally responsive teaching, use of active learning, and efforts directed at changing departmental culture.

The conference generated a great deal of energy and excitement, a sense that real change is happening. And yet, to many this activity seems reminiscent of the Calculus Reform efforts of the late 1980s and early '90s. I am often asked what is different now. Is this simply another iteration of a doomed effort to change these pivotal courses?

It is important to recognize that the Calculus Reform effort was not a failure. It made a real difference as can be seen by comparing textbooks of the 1980s and today. It emphasized the importance of graphical representations, verbal descriptions and communication skills, as well as projects and deep explorations of selected topics. It also served to initiate or accelerate efforts that are bearing fruit today such as Project NExT, the Scholarship of Teaching and Learning, and the explosive expansion of scholarly research into undergraduate mathematics education.

Nevertheless, those who worked at the forefront of the Calculus Reform movement had a vision that has not been realized, a vision that lives on in our current efforts. The goal is of calculus classes that engage all students in the joy of mathematical exploration and the satisfaction of deep learning, not just the memorization of procedures but the ownership of them so that their principles can be applied flexibly in unfamiliar situations.

Development of the current curriculum

Before explaining what is different this time around, it is useful to place the Calculus Reform movement into the context of the development of the modern undergraduate mathematics curriculum. As Alan Tucker has documented (Tucker, 2013), in the early 1950s undergraduate offerings in mathematics were meager and highly variable from one institution to the next. This decade would see the development of several different approaches to the first year of mathematics instruction. MAA produced *Universal Mathematics*, a broad overview of functions, limits, elements of calculus, sets, logic, counting, and probability. John Kemeny and colleagues at Dartmouth reworked this into *Introduction to Finite Mathematics*, covering logic, discrete probability, basic matrix algebra, Markov chains, and linear programming.

The launch of Sputnik in 1957 concentrated national attention on the need to produce many more and highly qualified engineers, scientists, and mathematicians. The National Science Foundation devoted considerable resources to the MAA's Committee on the Undergraduate Program in Mathematics (CUPM), which established its headquarters at Berkeley and enlisted many of the leading research mathematicians of the time to consider all aspects of undergraduate mathematics education. Their deliberations led to a series of influential reports.

Foremost among these was the *General Curriculum in Mathematics for Colleges* (CUPM, 1965, summarized by Duren, 1965). The curriculum it described for mathematically intensive majors began with three semesters of single and several variable calculus that included differential equations and a semester of linear algebra (a new course based on Kemeny's *Finite Mathematics* that few colleges offered before then). It cemented calculus as the focal point of the entire mathematics curriculum, the course toward which high school mathematics was directed and upon which all further mathematics courses would build.

This curriculum, supported by the expansion of public universities and the urge to funnel the best prepared students toward science, engineering, and mathematics, proved to be enormously successful. The number of PhDs awarded in mathematics rose from 300 in 1960 to 1200 in 1970, far more than the market could absorb. (I was then an undergraduate majoring in mathematics and was warned not to pursue a PhD as there would be no job for me.) Over the same decade, the number of Bachelor's degrees in mathematics and statistics grew from 11,400 in 1960 to 27,400 in 1970 (NCES, 2018, table 325.65). And then the bottom dropped out.

The growing discontent of the 1980s

By 1981, Bachelor's degrees in mathematics and statistics had fallen to just over 11,000, fewer than in 1960. Much of that loss can be explained by the rise in programs offering undergraduate degrees in computer science, from 2400 degrees in 1971 to 15,000 in 1981, on its way to a local maximum of 42,000 in 1986 (NCES, 2018, table 325.25). CUPM produced a report on the General Mathematical Sciences Program in 1981 (reprinted in Steen, 1989), acknowledging the problem of the drop in mathematics majors and pointing to the need for mathematics programs to consider the needs of future computer scientists. CUPM identified five philosophical principles that should undergird the major. The second was that the curriculum should be "designed around the abilities and academic needs of the average mathematical sciences student (with supplementary work to attract and challenge talented students.)." The third was to "use interactive classroom teaching to involve students actively in the development of new material."

In 1984, Tony Ralston drew on the shift toward computational science and its reliance on discrete mathematics to argue that "the time has come for discrete mathematics to play a coequal role with calculus in the first two years of college mathematics" (Ralston, 1984, p. 373). The reactions were swift and vociferous. Most of the mathematical community lined up behind the position that calculus should retain its pre-eminence in the undergraduate curriculum. But one of Ralston's most outspoken critics, Ron Douglas, then chair of the mathematics department at SUNY Stony Brook and soon to become Dean of Science and then Provost, realized that trying to defend calculus as taught was an impossible task.

Douglas organized a series of national gatherings that managed to shift attention from the debate between discrete and continuous mathematics to issues of the how and what of calculus instruction. As he wrote for the Tulane Conference of 1986,

> The United States is currently experiencing a shortage of young people studying mathematics, science and engineering, and this shortage is expected to worsen. Calculus is the gateway and is fundamental to all such study. Hence every student who does not complete calculus is lost to further study in science, mathematics or engineering. Moreover, many students who start calculus do not complete it successfully. The country cannot afford this now, if it ever could. Further, many of those who do finish the course, have taken a watered down, cookbook course in which all they learn are recipes, without even being taught what it is that they are cooking. Understandably, science and engineering faculties find it difficult to build on such a foundation, and they feel that they must teach their students elementary calculus as well as science or engineering. Finally, in past generations many students were sufficiently challenged and turned-on in their calculus course that they decided to become mathematicians. I don't believe that happens much today. To overcome these problems and to recapture that earlier excitement, I decided to see what could be done to improve calculus instruction. (Douglas, 1986, p. iv)

The solution that he proposed was the creation of a revised curriculum that would decrease the number of topics and instead emphasize the role of calculus in "modeling and understanding the real world." Calculus Reform was born.

The Calculus Reform movement

Those working on this effort soon realized that it would require more than a new curriculum. The National Science Foundation was enlisted, launching the first grants of the NSF Calculus Initiative in 1988. As Lida Barrett and William Browder wrote in their report to NSF in 1989,

> The contemplated changes are not in the overall content of the course, but rather in the expectation of better student understanding; in great stress on applications and links with other disciplines; in the utilization of numerical methods and computer techniques; and in encouraging a fresh approach to teaching. (as quoted in Tucker and Leitzel, 1995, p. 16)

Over the next seven years, the NSF Calculus Initiative handed out over $22 million in 127 grants that spurred the development of textbooks and programs. There are no broad new ideas today that were not piloted during these peak years of Calculus Reform, 1988–95. This included active learning pedagogies, experimentation with uses of technology, group work, an emphasis on projects and writing, and the incorporation of applications. To accomplish these changes, much of the emphasis, especially the early emphasis, was on new curricula.

The modifications of the curriculum opened this effort to criticism. Among the innovations introduced at this time were trimmed lists of topics, less emphasis on algebraic manipulation, and heavy reliance on computers or graphing calculators. There was already concern over the

NCTM Standards (NCTM, 1989) that many interpreted as lowering expectations for what students should learn in high school. Calculus Reform was often viewed as encouraging this trend. A few critics went so far as to assert that high failure rates were desirable because of the message they sent to the high schools.

By the mid-90s, the impetus behind Calculus Reform was weakening. The NSF Calculus Initiative ended in 1995, as did publication of *UME Trends*, the joint publication of AMS, MAA, and SIAM that had begun in 1988 to share news of efforts to reform undergraduate mathematics education. The sense of urgency to reform had been lost. The trajectory of the calculus innovations is illustrated by three graphs that I published in my *Launchings* column of June, 2007, Reform Fatigue (Figures 1–3).

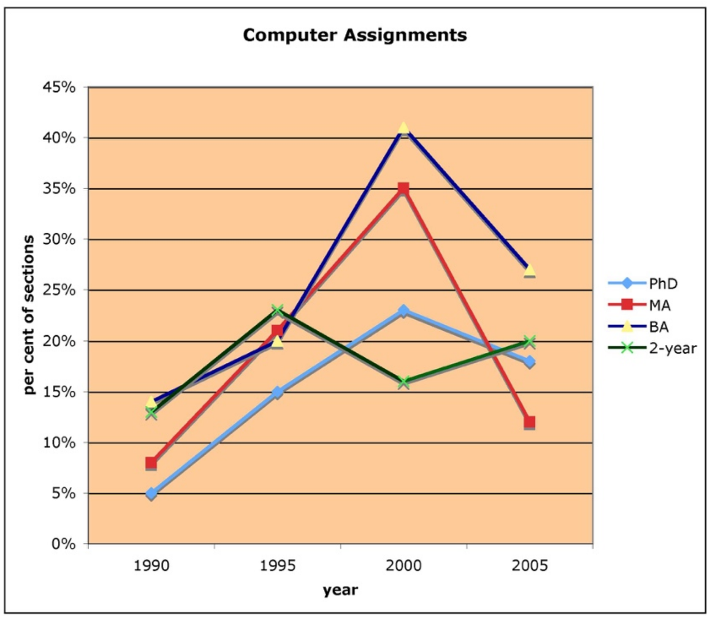

Figure 1: Percentage of Calculus I courses using computer assignment.
Departments classified by highest degree offered in mathematics.
Source: CBMS Statistical Abstracts for 1990, 1995, 2000, and 2005.

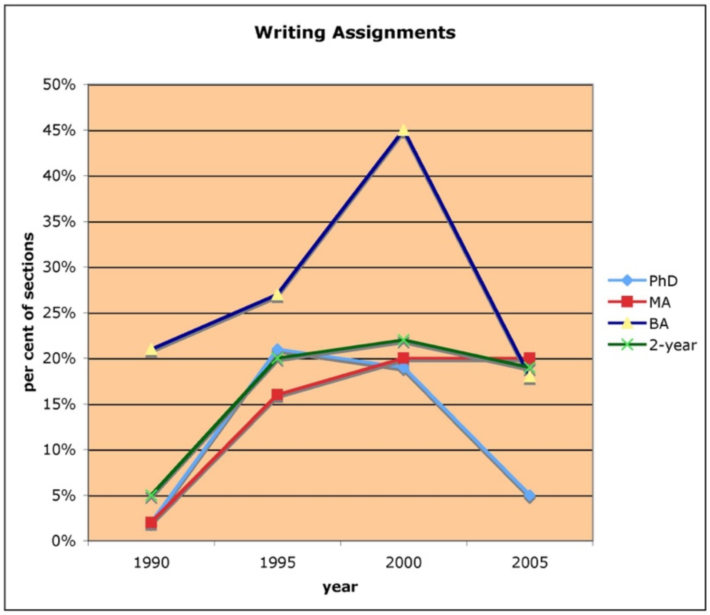

Figure 2: Percentage of Calculus I courses using writing assignment. .
Departments classified by highest degree offered in mathematics.
Source: CBMS Statistical Abstracts for 1990, 1995, 2000, and 2005.

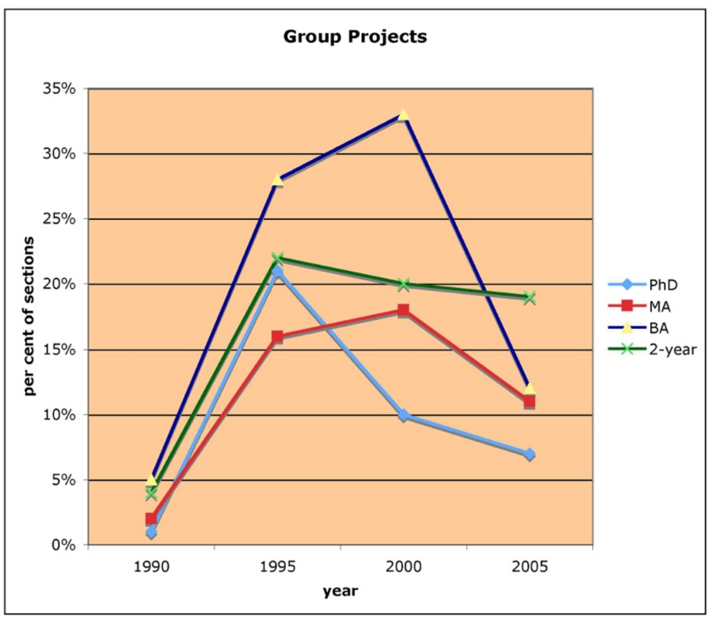

Figure 2: Percentage of Calculus I courses using group projects. .
Departments classified by highest degree offered in mathematics.
Source: CBMS Statistical Abstracts for 1990, 1995, 2000, and 2005.

For writing assignments and group projects, two of the signal thrusts of Calculus Reform, we see that the research universities (those offering a PhD in mathematics) went from almost none in 1990 to about a fifth in 1995, then support dropped off, collapsing by 2005, after which CBMS stopped recording these percentages. Efforts at liberal arts colleges continued to climb until 2000, but dropped after that. For no group did the penetration reach 50%.

The aftermath

Work on improving undergraduate mathematics education continued, but it had moved into a less aggressive phase with a much lower profile. NSF still funded promising efforts, initially through *Course, Curriculum, and Laboratory Improvement* (CCLI) started in 1998 and subsuming an earlier program on *Course & Curriculum Development* (CCD). CCLI became *Transforming Undergraduate Education in STEM* (TUES) in 2010. *Widening Implementation & Demonstration of Evidence-Based* Reforms (WIDER) was added in 2011. In 2013, these were merged with the *STEM Talent Expansion Program* (STEP) to form *Improving Undergraduate STEM Education* (IUSE), the current program under which both PtC and SEMINAL are funded.

Project NExT began in 1994, sharing strategies for the improvement of undergraduate mathematics education with a cohort of 80 to 90 new doctoral recipients each year. MAA's special interest group on Research in Undergraduate Mathematics Education began in 1999, both reflecting and spurring dramatic growth in the community of those engaged in research in this area. That same year, AMS published *Towards Excellence* (Ewing, 1999), making the point that a healthy research department must also be concerned about undergraduate education and illustrating examples of good undergraduate teaching. PRIMUS, founded in 1991 to publish work in the scholarship of teaching and learning for undergraduate mathematics, continued to flourish.

The new century saw an expansion of interest in and research around Inquiry Based Learning as well as the development of courses focused on quantitative reasoning. In 2004, CUPM published the next iteration of its curriculum guides (CUPM, 2004), presenting recommendations that built on earlier CUPM advice and encouraging the active and engaged learning that Calculus Reform had tried to facilitate. For the first time, it focused on undergraduate mathematics in service to other departments, especially in the STEM fields, building on the Curriculum Foundations Project (Ganter and Barker, 2004) that MAA had launched to better understanding the needs of these partner disciplines. The year 2009 saw the start of work by the Carnegie Foundation and the Charles A. Dana Center on what would become the Pathways projects, providing alternate routes into the mathematical sciences for students who were not well prepared for college algebra. In 2010, MAA began its series of studies of calculus instruction. At about the same time, the National Academies' Board on Mathematical Sciences and their Applications (BMSA, today standing for the Board on Mathematical Sciences and Analytics) began its work on *The Mathematical Sciences in 2025* (NRC, 2013), illustrating the role of mathematics in the creation of innovative science and technology and arguing for an undergraduate curriculum that both celebrates these connections and prepares students for new roles in the mathematical sciences.

The period 1995 to 2012 thus saw a great deal of activity, but little of it was visible outside the community of those working on these issues. This point was driven home forcefully when the President's Council of Advisors on Science and Technology released its 2012 report, *Engage to Excel* (PCAST, 2012), castigating the mathematics community for its apparent failure to work on undergraduate mathematics instruction or to employ what by now were recognized to be evidence-based teaching strategies, active learning in particular.

Largely in response to this criticism, Phillip Griffiths, Eric Friedlander, Uri Treisman, and Mark Green met with the leadership of the Carnegie Corporation of New York to launch *Transforming Post-Secondary Education in Mathematics* (TPSE-Math). Although working across all post-secondary institutions, TPSE-Math has leveraged the reputations of its leaders as research mathematicians to engage top university departments of mathematics. Other efforts designed to increase visibility both within and beyond the mathematics community have included the *Common Vision* report (Saxe and Braddy, 2015), jointly sponsored by AMATYC, AMS, ASA, MAA, and SIAM; the MAA's Instructional Practices Guide (Abell et al., 2018), APLU's SEMINAL project designed to help public universities transform their calculus sequence, and the joint statement of the presidents of the CBMS societies urging all departments of mathematics to embrace active learning (CBMS, 2016). CBMS presidents have defined active learning as "classroom practices that engage students in activities, such as reading, writing, discussion, or problem solving, that promote higher-order thinking" and have called on

> institutions of higher education, mathematics departments and the mathematics faculty, public policy-makers, and funding agencies to invest time and resources to ensure that effective active learning is incorporated into post-secondary mathematics classrooms. (CBMS, 2016)

What *is* different this time?

In 1997, *The Chronicle of Higher Education* published its post-mortem of the Calculus Reform movement. The article concluded with a discouraging comment from Ed Dubinsky, one of the fathers of this effort, "Except for a small number of isolated pockets, it will be hard to tell that there was a calculus reform. [In a few years] we'll become upset that very few people are really learning calculus and we'll have another round of reforms. I hope that round survives."

I see three reasons why this is not just a repeat of what happened thirty years ago.

First, the reform agenda that drove Calculus Reform did not disappear. Rather, it developed a lower profile that saw an acceleration of undergraduate mathematics research and a continuation of experimentation leading to a better understanding of the critical elements for improved undergraduate instruction. Today we are building on thirty years of experience. We have a more sophisticated sense of what works and what does not, of where technology can be a support and where it is a hindrance. We have accumulated data to back up assertions of best practice. Part of this building process has involved making connections to educational researchers in other STEM fields, especially the physics education research community, but also in biology, chemistry, geology, and engineering education.

Second, in 1990 the argument could be made that undergraduate mathematics education was working. The 1965 CUPM report on the general curriculum was directed toward students who would be ready to start calculus when they got to college, predominantly middle class White males. In the 1960s, they constituted over half of college graduates. While the percentage of Bachelor's degrees going to White males had dropped to 38% by 1990, we were still producing adequate numbers of scientists and engineers who were able to meet the challenges they would face. Moreover, the experiments of the early Calculus Reform could and sometimes did go wrong. Departments were often reluctant to run the risk of changing the approaches then in place. Adding to this reluctance was the widespread belief that whatever was wrong was the fault of K-12 education, not the colleges and universities.

Today, the flaws of traditional methods for teaching calculus are far more apparent. The reaction to Calculus Reform that asserted high failure rates to be the price of improving K-12 education is now totally unacceptable. Presidents, provosts, and deans have come to recognize the cost to their institutions of high failure rates. There is pressure both to improve passing rates *and* to ensure those students go on to succeed in their subsequent courses. In too many cases, the status quo accomplishes neither. This is the economic argument for embracing changes that we know work. Such economic imperatives carry a lot of weight.

Combined with these pressures is the recognition that White males now barely exceed a quarter of college graduates, and demands for the employ of the mathematical sciences have been expanding and changing in fundamental ways. ASA's GAISE report for undergraduate statistics (ASA, 2016), COMAP and SIAM's GAIMME report (COMAP and SIAM, 2019), and especially *The Mathematical Sciences in 2025* (NRC, 2013) make it clear that today's undergraduate preparation in mathematics must be more than a proving ground where students demonstrate that they can survive the curriculum of the 1960s. It must actually begin the process of equipping them for the challenges they will face in the changing landscape of 21st century workforce demands.

Third, the goals this time around are very different. There was a naiveté to the Calculus Reform movement, believing that if we built the ideal calculus curriculum then mathematicians would embrace it and adapt to the demands it made on how they taught calculus. Today the focus is on training and support for new generations of educators. We are learning that we must demonstrate how to promote student engagement in the kind of learning that will lead to ownership of the concepts and methods. We are now learning what it takes to prepare and support graduate students and new faculty as well as experienced faculty to teach in this way.

While a bit simplistic, the third thing that is different this time can be summarized as a shift from an emphasis on what is taught to how it is taught. This is combined with the recognition that teaching for meaningful learning is not easy. Building the structures that support it requires buy-in from the dean of science, the department chair, a core of senior faculty, and one's colleagues both in the department and beyond.

Calculus Reform was not a movement that came and went. It was the opening of a multi-decadal effort that only now is truly beginning to blossom.

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Based on NCES data on graduating class of 2017, White non-Hispanic males make up

27% of all majors
23% of bio majors
33% of math and stat majors
41% of physical science majors
47% of computer science majors
48% of engineering majors