Issues of the Transition to College Mathematics

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PowerPoint available at www.macalester.edu/~bressoud/talks
1. Data on enrollment numbers and majors in mathematics.

2. Why calculus in high school is a pressing issue.

3. What we know and need to know about the transition from high school to college mathematics.

4. The nature of high school calculus and how it needs to change.

5. First-year college mathematics and how it needs to change.
Percentage of students in 4-year undergraduate programs enrolled in mathematics at level of calculus or above (Fall term)

Denominator = # of FTE students enrolled increased by 43% from 1985 to 2005

CBMS and NCES data
Number of students (thousands) in 4-year undergraduate programs enrolled in mathematics at level of calculus or above (Fall term)

Total population of the US increased by 25% from 1985 to 2005

CBMS data
Number of students (thousands) in 4-year undergraduate programs enrolled in mathematics at level of calculus or above (Fall term)

- Research Universities
- undergrad/comprehensive

CBMS data
Mathematics & Statistics

Source: American Freshman and NCES
Research universities as percent of all Bachelor’s Degrees in Mathematical Sciences:
Calculus enrollments (thousands) by type of institution

<table>
<thead>
<tr>
<th>Year</th>
<th>Research Univ</th>
<th>Comprehensive Univ</th>
<th>Undergrad College</th>
<th>2-year College</th>
</tr>
</thead>
<tbody>
<tr>
<td>1990</td>
<td>193.4</td>
<td>71.9</td>
<td>107.4</td>
<td>90</td>
</tr>
<tr>
<td>1995</td>
<td>160</td>
<td>72</td>
<td>105</td>
<td>95</td>
</tr>
<tr>
<td>2000</td>
<td>182</td>
<td>73</td>
<td>96</td>
<td>84</td>
</tr>
<tr>
<td>2005</td>
<td>210</td>
<td>51</td>
<td>98</td>
<td>81</td>
</tr>
</tbody>
</table>
137,000 prospective engineers this past fall
275,000 students in fall calculus?
30% increase over 2005?

CBMS and CIRP data
Conclusions:

1. In undergraduate colleges and comprehensive universities, we have been losing math students at the level of calculus and above as well as majors.

2. In research universities, the number of students studying mathematics at the level of calculus and above as well as the number of majors is highly correlated with the number of incoming students expecting to major in engineering.
Conclusions (continued):

3. We survive by attracting students to the mathematics major *after* they enter college.

4. To do this, we must pay attention to
   a) the number of first-year students studying mathematics in our colleges and universities at the level of calculus, and
   b) the number of these first-year students who are enticed to pursue the study of mathematics.
Figure 1. Percentage of high school graduates who completed different levels of mathematics courses: 1982, 1992, and 2004

- No math or low academic math
- Algebra I/plane geometry
- Algebra II
- Algebra III/trigonometry/analytic geometry
- Precalculus
- Calculus

Year

1982
- 24.9
- 30.6
- 18.2
- 15.6
- 4.8
- 5.9

1992
- 12.9
- 22.6
- 26.4
- 16.4
- 11.0
- 10.7

2004
- 5.2
- 18.1
- 25.7
- 18.0
- 18.9
- 14.1

$\frac{3}{4} \geq \text{Algebra II} \quad \frac{1}{3} \geq \text{Precalculus}$

550,000–600,000 students studied calculus in high school this year, roughly 1/3 of the 1.8 million who will go directly from HS to college.

Between 150,000 and 200,000 of these students earn and take advantage of credit for Calculus I.
Fall Enrollments, Calculus II (thousands)

CBMS data
Since 1995, there has been a **22% decrease** in the number of students taking Calculus II in the Fall term at comprehensive universities, undergraduate colleges, and 2-year colleges.

CBMS data
We need to:

Get more information about what happens to students who study calculus in high school.

Establish and enforce guidelines for high school programs offering calculus.

Re-examine first-year college mathematics.
Morgan & Klaric, 2007: study of 22 colleges and universities in fall, 1994; grades weighted so that SAT scores are comparable

<table>
<thead>
<tr>
<th>Placed via</th>
<th>average grade in Calculus II</th>
<th>SAT Adjusted grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>Passed Calculus I</td>
<td>2.43</td>
<td></td>
</tr>
<tr>
<td>3 on AB exam</td>
<td>2.69</td>
<td>2.64</td>
</tr>
<tr>
<td>4 on AB exam</td>
<td>2.90</td>
<td>2.78</td>
</tr>
<tr>
<td>5 on AB exam</td>
<td>3.34</td>
<td>3.15</td>
</tr>
</tbody>
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<th>Placed via</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Passed Calculus I</td>
<td>2.50</td>
<td></td>
</tr>
<tr>
<td>3 on BC exam</td>
<td>3.00</td>
<td>2.92</td>
</tr>
<tr>
<td>4 on BC exam</td>
<td>3.45</td>
<td>3.35</td>
</tr>
<tr>
<td>5 on BC exam</td>
<td>3.46</td>
<td>3.27</td>
</tr>
</tbody>
</table>

Keng & Dodd, 2008: study at UT-Austin, 1998–2001; students selected by stratified random sample so that SAT distribution matched that of AP students.

<table>
<thead>
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<th>Preparation for Calculus II</th>
<th>Average grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) 3 or higher on BC exam</td>
<td>3.43</td>
</tr>
<tr>
<td>b) Passed Calculus I, SAT matched to BC students</td>
<td>3.16</td>
</tr>
<tr>
<td>c) 3 or higher on AB exam</td>
<td>3.13</td>
</tr>
<tr>
<td>d) Passed Calculus I, SAT matched to AB students</td>
<td>3.03</td>
</tr>
<tr>
<td>e) 3 or higher on AB exam and took Calculus I</td>
<td>2.96</td>
</tr>
<tr>
<td>f) Dual enrollment credit</td>
<td>2.93</td>
</tr>
<tr>
<td>g) BC course, no credit, took Calculus I</td>
<td>2.82</td>
</tr>
<tr>
<td>h) AB course, no credit, took Calculus I</td>
<td>2.45</td>
</tr>
</tbody>
</table>

Statistically significant at 0.05 all four years:
a) over b)
two of four year: c) over d)
one of four year: c) over e)
Two Current Studies:

1. Phil Sadler, Harvard, *Factors Influencing College Success in Mathematics*
   HS factors that influence success in Calculus I

2. MAA (Bressoud, Carlson, Pearson, Rasmussen), *Characteristics of Successful Programs in College Calculus*
   College factors that influence success in Calculus I and case study analysis of successful programs.
Of the high school students who graduated in 1992 and studied “calculus” while in high school, 31% took *precalculus* in college, and a further 32% took *no calculus* in college.

The anecdotal evidence is strong that today’s students who do not qualify for college credit (roughly 400,000) struggle to articulate high school and college mathematics.
We need to:

Get more information about what happens to students who study calculus in high school.

Establish and enforce guidelines for high school programs offering calculus.

Re-examine first-year college mathematics.
Trying to restrict the growth of AP Calculus is a non-starter:

• Success in AP Calculus is the single most useful predictor of successful completion of college.

• Students without access to a good program in AP Calculus are at a competitive disadvantage.

• ExxonMobil and the Gates Foundation are strongly behind the spread of quality programs in AP Calculus.
Several states, including Hawaii, Mississippi, South Carolina, and Tennessee, have state high school standards that describe a watered down version of calculus.

California’s standards prescribe a course that is equivalent to a full year of college calculus.

No state standards talk about the goals of the course or prescribe pre-requisite knowledge.
We need to:

Get more information about what happens to students who study calculus in high school.

Establish and enforce guidelines for high school programs offering calculus.

Re-examine first-year college mathematics.
2001–06 study at Arizona State University

Of students who took pre-calculus and
• Their declared major required at least one semester of calculus, and
• They earned an A in pre-calculus,

43% chose *not* to enroll in calculus.
During the period fall 2001 through fall 2006,

43% of engineering majors,

54% of mathematics majors,

51% of physical science majors, and

50% of technology majors

who enrolled in Calculus I at ASU and whose intended majors required Calculus II never earned credit for Calculus II.
US Military Academy, West Point:

MA 103: Mathematical Modeling and Introduction to Calculus, the first of our core courses for all cadets. This is the first of four courses in the USMA mathematics core curriculum. The focus of the course is to use effective problem solving and modeling techniques to find solutions to complex and often ill-defined problems. The course lays the foundation for calculus and differential equations through difference equations.

MA 104: Calculus I. This course builds upon the foundation laid in MA103, as the cadet learns about differential calculus in single and multi-variable problems.

This program has now been in place for twenty years. A similar program at Macalester is over 5 years old.
Competency E1

Apply quantitative reasoning and appropriate mathematics to describe or explain phenomena in the natural world.
Learning Objectives:

1. Demonstrate quantitative numeracy and facility with the language of mathematics.
2. Interpret data sets and communicate those interpretations using visual and other appropriate tools.
3. Make statistical inferences from data sets.
4. Extract relevant information from large data sets.
5. Make inferences about natural phenomena using mathematical models.
6. Apply algorithmic approaches and principles of logic (including the distinction between cause/effect and association) to problem solving.
7. Quantify and interpret changes in dynamical systems.
“The mathematics profession as a whole has seriously underestimated the difficulty of teaching mathematics.”

Ramesh Gangolli
MER Workshop
May 31, 1991

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