Issues of the Transition to College Calculus

David Bressoud
Macalester College, St. Paul, MN

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PowerPoint available at
www.macalester.edu/~bressoud/talks
1. Data on graduation rates in math-intensive majors.

2. Data on high school and college mathematics enrollments.

3. The effect of calculus in high school.

4. What needs to change in our colleges and universities.
Remarks by President Obama at the National Academy of Sciences, April, 2009:

“... our work does not end with a high school diploma. For decades, we led the world in educational attainment, and as a consequence we led the world in economic growth ... But in this new economy, we've come to trail other nations in graduation rates, in educational achievement, and in the production of scientists and engineers.”
Hispanic students as % of Bachelors Degrees in selected fields

NCES data
There were 1,089 Bachelors in Math or Stat earned by African-Americans in 1997. By 2008, that number was down to 818.
In the Fall of 2010: 210,000 students entered four-year undergraduate programs with the intention of majoring in engineering, a physical science, mathematics, or statistics. About half will succeed.

Over 600,000 of the entering students had passed a class in calculus while in high school.
Figure 1. Percentage of high school graduates who completed different levels of mathematics courses: 1982, 1992, and 2004

- 24.9% completed Precalculus
- 30.6% completed Algebra II
- 18.2% completed Algebra I/Plane geometry
- 15.6% completed Algebra II
- 4.8% completed Algebra III/Trigonometry/Analytic geometry
- 5.9% completed no math or low academic math

Over 600,000 students studied calculus in high school this year, roughly 1/3 of the 1.8 million who will go directly from HS to college.

Slightly over 150,000 of these students earned and used credit for Calculus I.
Fall Enrollments, Calculus II (thousands)

4-year colleges
2-year colleges

CBMS data
Fall enrollments (thousands) in 2-year undergraduate programs

<table>
<thead>
<tr>
<th>Year</th>
<th>Precollege</th>
<th>Introductory</th>
<th>Calc &amp; Advanced</th>
</tr>
</thead>
<tbody>
<tr>
<td>1985</td>
<td>482</td>
<td>294</td>
<td>113</td>
</tr>
<tr>
<td>1990</td>
<td>724</td>
<td>361</td>
<td>140</td>
</tr>
<tr>
<td>1995</td>
<td>800</td>
<td>415</td>
<td>142</td>
</tr>
<tr>
<td>2000</td>
<td>763</td>
<td>396</td>
<td>117</td>
</tr>
<tr>
<td>2005</td>
<td>964</td>
<td>501</td>
<td>120</td>
</tr>
</tbody>
</table>

45% increase in 2-year college enrollments during this time

CBMS data
Fall enrollments (thousands) in 4-year undergraduate programs

<table>
<thead>
<tr>
<th>Year</th>
<th>precollege</th>
<th>introductory</th>
<th>calculus level</th>
<th>advanced</th>
</tr>
</thead>
<tbody>
<tr>
<td>1985</td>
<td>251</td>
<td>593</td>
<td>637</td>
<td>138</td>
</tr>
<tr>
<td>1990</td>
<td>261</td>
<td>592</td>
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<td>119</td>
</tr>
<tr>
<td>1995</td>
<td>222</td>
<td>613</td>
<td>538</td>
<td>96</td>
</tr>
<tr>
<td>2000</td>
<td>219</td>
<td>723</td>
<td>570</td>
<td>102</td>
</tr>
<tr>
<td>2005</td>
<td>201</td>
<td>706</td>
<td>587</td>
<td>112</td>
</tr>
</tbody>
</table>

43% increase in 4-year college enrollments during this time

CBMS data
Fall enrollments (thousands), Calculus and above, 4-year undergraduate programs

CBMS data
Increasingly, the students who do not have to take mathematics in college, don’t.
Morgan & Klaric, 2007: study of 22 colleges and universities in fall, 1994; grades weighted so that SAT scores are comparable

<table>
<thead>
<tr>
<th>Placed via</th>
<th>average grade in Calculus II</th>
<th>SAT Adjusted grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>Passed Calculus I</td>
<td>2.43</td>
<td></td>
</tr>
<tr>
<td>3 on AB exam</td>
<td>2.69</td>
<td>2.64</td>
</tr>
<tr>
<td>4 on AB exam</td>
<td>2.90</td>
<td>2.78</td>
</tr>
<tr>
<td>5 on AB exam</td>
<td>3.34</td>
<td>3.15</td>
</tr>
</tbody>
</table>

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</tr>
</thead>
<tbody>
<tr>
<td>Passed Calculus I</td>
<td>2.50</td>
<td></td>
</tr>
<tr>
<td>3 on BC exam</td>
<td>3.00</td>
<td>2.92</td>
</tr>
<tr>
<td>4 on BC exam</td>
<td>3.45</td>
<td>3.35</td>
</tr>
<tr>
<td>5 on BC exam</td>
<td>3.46</td>
<td>3.27</td>
</tr>
</tbody>
</table>


Students who earned 3 or higher on AB exam and choose to retake Calculus I did **worse** in Calculus II then those who went directly to Calculus II.

*Caveats*: Difference was statistically significant at .05 only 1 out of 4 years.

Not controlled for comparability of ability levels
Those who do not have access to calculus in high school are at a serious disadvantage.

All evidence suggests that calculus in high school works well for most of the roughly 25% who earn and use their college credit.

What about the other 75%?
Controlling for socio-economic factors, what aspects of high school mathematics prepare students for success in Calculus I?
Numerous studies (Sadler and others) that control for ability (SAT/ACT) and socio-economic factors (e.g. parental education):

Students who study Calculus in HS and do well on placement exam ($3^+$ on AB exam) do significantly better in Calculus I as well as intro Biology, Chemistry, and Physics.

There is little or no discernible benefit from simply taking Calculus in High School.
Bressoud, Carlson, Pearson, Rasmussen: *Characteristics of Successful Programs in College Calculus*

College factors that influence success in Calculus I and case study analysis of successful programs.
Random sample of 12,000 mainstream Calculus I students at all types of colleges and universities across the US (out of about 250,000):

• 61% took a calculus course in high school
  • 54% of these took an AP Calculus exam
    • 60% of those who took the AP Calculus exam earned a 3 or higher (94 students in the sample earned 5 on the BC exam)

• 10% are taking calculus in college for at least the second time
Of the high school students who graduated in 1992 and earned credit for “calculus” while in high school, 31% took *precalculus* in college, and a further 32% took *no calculus* in college.

Of the high school students who graduated in 2004 and earned credit for “calculus” while in high school, 17% took *remedial mathematics* in college.

NCES, NELS:88 and ELS:2002/06 data.
Of the high school students who graduated in 1992 and earned credit for “calculus” while in high school, 31% took precalculus in college, and a further 32% took no calculus in college.

We must have clear, enforced guidelines for what it means to be ready for calculus in high school.

Of the high school students who graduated in 2004 and earned credit for “calculus” while in high school, 17% took remedial mathematics in college.

NCES, NELS:88 and ELS:2002/06 data.
Of students who took pre-calculus and
• Their declared major required at least one semester of calculus, and
• They earned an A in pre-calculus,

43% chose *not* to enroll in calculus.
During the period fall 2001 through fall 2006, 43% of engineering majors, 54% of mathematics majors, 51% of physical science majors, and 50% of technology majors who enrolled in Calculus I at ASU and whose intended majors required Calculus II never earned credit for Calculus II.
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The point is that ASU gathered this information, and they are now doing something about it.
SOLUTIONS: Improve first-year college mathematics

Use online resources to address individual student weaknesses.

Alison Ahlgren
MA 103: Mathematical Modeling and Introduction to Calculus.
The course lays the foundation for calculus and differential equations through difference equations and dynamical systems.

This course has now been in place for twenty years.
A similar course at Macalester is over 5 years old.
MTBI supports the development of students through educational, research and mentorship activities from the undergraduate to the postdoctoral level.

Carlos Castillo-Chavez
“The mathematics profession as a whole has seriously underestimated the difficulty of teaching mathematics.”

Ramesh Gangolli
MER Workshop
May 31, 1991

With thanks to Susanna Epp for preserving this quote.

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