Characteristics of Successful Programs In College Calculus
A report on our findings

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PowerPoint available at
www.macalester.edu/~bressoud/talks

Iowa State University
Ames, IA
March 30, 2012
August 2011,
Intel CEO Paul Otellini:

“Looking forward, this nation is at risk of a significant shortfall of qualified experts in science and math to meet the country’s needs.”
President's Council of Advisors on Science and Technology (PCAST)

February, 2012
Recommendations:

1. Catalyze widespread adoption of empirically validated teaching practices.
2. Advocate and provide support for replacing standard laboratory courses with discovery-based research courses.
3. Launch a national experiment in postsecondary mathematics education to address the mathematics-preparation gap.
4. Encourage partnerships about stakeholders to diversify pathways to STEM careers.
210,000 students per year enter four-year undergraduate programs intending to major in engineering, a physical science, mathematics, or statistics.

An additional 300,000 are enrolled in one of these programs in a two-year college.

We graduate 107,000 with a Bachelor’s degree in one of these disciplines.
Bachelors Degrees, math-intensive majors

NCES data
Math-intensive Bachelor's degrees relative to number of 22 year-olds (degrees ÷ # of 22-year olds)

- Engineering
- Physical Science
- Math & Stat

NCES & US Census data
Math & Stats Majors by Gender

NCES data
Hispanic American as % of Total Degrees

- Degrees in Engineering
- Degrees in Mathematics
- Degrees in Physical Sciences

NCES data
There were 1,089 Bachelors in Math or Stat earned by African-Americans in 1997. By 2009, that number was down to 876.
MAA Study of Calculus

*Characteristics of Successful Programs in College Calculus*

NSF EHR DRL #0910240
Five years: 2009–2014

PI: David Bressoud
Co-PI’s: Marilyn Carlson, ASU
        Vilma Mesa, U Michigan
        Michael Pearson, MAA
        Chris Rasmussen, SDSU
Phase I: Survey
Aug. 2009 – July 2010, preparation of instruments, recruitment of institutions

1. Institutional questionnaire, chair
2. Calculus coordinator questionnaire
3. Instructor pre-term survey
4. Student 2\textsuperscript{nd} to 3\textsuperscript{rd} week survey
5. Student 2\textsuperscript{nd} to 3\textsuperscript{rd} last week survey
6. Instructor post-term survey
7. Collection of final exams and grades

Feb. 2011 onward, analysis of data
Phase II: Case Studies
Feb. 2011 – Aug. 2012, recruitment of teams, preparation of protocols, recruitment of institutions

Sept. – Nov. 2012, case study visits
Each of the four teams will make a 2–3 day visit to four institutions

Dec. 2012 onward, analysis of data, publication of results

July 2014, end of grant
People:

Marilyn Carlson, ASU, coordinator for survey

Chris Rasmussen, SDSU, coordinator for case studies

Phil Sadler & Gerhard Sonnert, Harvard, Statistical Analysis

Peter Ewell, NCHEMS, consultant and external evaluator

Michael Pearson and Olga Dixon, MAA, program administration
Case Study Team Leaders:

Chris Rasmussen, SDSU, coordinator for case studies, doctoral universities

Eric Hsu, SFSU, master’s universities

Sean Larsen, PSU, bachelor’s colleges

Vilma Mesa, U Michigan, two-year colleges
168 colleges and universities

660 instructors representing almost 900 Calculus I classes and over 34,000 students

12,000 students answered the initial survey
Calculators allowed for at least some exam questions

Graphing calculator: 90% in high school, 50% in college
Calculator with CAS: 60% in high school, 20% in college
61% all Calculus I students took a calculus class in high school. 61% of them earned an A (37% of all Calc I students)

For 69% of those took Calc in HS, it was an AP Calculus course (42% of all Calc I students).

81% of the AP Calculus students took the AP exam (34% of all Calc I students)

60% of those who took the exam earned a 3 or higher (just over 20% of all Calc I students)
11.4% of all Calc I students had earned a 4 or higher on the AB exam or a 3 or higher on the BC exam.

0.9% earned a 5 on the BC Calculus exam.

1.7% took AB Calculus one year, BC Calculus the following year, and then took Calculus I when they got to college. Extrapolates to over 5,000 such students in Fall Calculus I each year.
These are good students:

- Average SAT Math: 652, standard deviation = 76, Interquartile range [610,700]
- 95% believe they have knowledge and abilities to succeed in calculus
- 89% find using reasoning to solve math problems a satisfying experience
- 83% enjoy mathematics
- 65% would be taking this course even if it were not required
They want to understand calculus:

74% prefer to make sense of the mathematics rather than simply memorizing it

72% see the role of the instructor as helping students to reason through problems on their own rather than showing students how to work the problems

58% expect to earn an A in this course
Grade for college Calculus I:

22% A

28% B

23% C

27% D, F, or Withdrew
Dependent Variables

Pre- and post-test:

• I am confident in my mathematical abilities
• If I had a choice, I would continue to take mathematics
• I enjoy doing mathematics
• I intend to take Calc II

Post-test only:

• This course has increased my interest in taking more mathematics
Control Variables

Demographics
   Gender, SES, Race/Ethnicity

HS Math
   Math courses taken; if calculus, what kind; if AP exam, which and what score
   Grade in last HS math course
   SAT/ACT scores

College
   Prior college math, year in college, career intention
   Pre-survey value
Independent Variables

Student Level:
• Student beliefs and attitudes about learning mathematics
• Study habits
• Level of intellectual engagement with the course
• Experience with technology (graphing calculators and/or computer software)
Independent Variables

Classroom Level—Instructor supplied:

• Class size
• Instructor experience and background
• Instructor beliefs, attitudes, and interests
• Assessment practices
• Out of class interactions with students
• Use of technology including use of web resources
• Textbook as well as additional instructional resources provided for students
Independent Variables

Classroom Level–student supplied:

• Student perceptions of instructional practices
• Use of technology
• Student perceptions of assessment practices
• The intellectual community outside of class
Independent Variables

Institutional Level:

- Placement procedures
- Technological support
- Institutional support for students
- Institutional support for instructors
55% men

97% full-time students

75% freshmen

76% White, 14% Asian, 5% Black, 10% Hispanic
Total college population: 73% White, 9% Asian, 12% Black, 12% Hispanic

75% intend to major in Science or Engineering
(Bio 30%, Eng 30%, Phys Sci 6%, CS 5%)
## Dependent Variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>SD</th>
<th>SE</th>
<th>Δ</th>
<th>ES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Confidence</td>
<td>3.89</td>
<td>1.01</td>
<td>0.02</td>
<td>3.42</td>
<td>1.18</td>
</tr>
<tr>
<td></td>
<td>3.42</td>
<td>1.18</td>
<td>0.02</td>
<td>-0.47</td>
<td>-0.46</td>
</tr>
<tr>
<td>Enjoyment</td>
<td>3.63</td>
<td>1.27</td>
<td>0.02</td>
<td>3.28</td>
<td>1.37</td>
</tr>
<tr>
<td></td>
<td>3.28</td>
<td>1.37</td>
<td>0.02</td>
<td>-0.35</td>
<td>-0.27</td>
</tr>
<tr>
<td>If I had a Choice...</td>
<td>1.93</td>
<td>1.02</td>
<td>0.02</td>
<td>1.84</td>
<td>1.08</td>
</tr>
<tr>
<td></td>
<td>1.84</td>
<td>1.08</td>
<td>0.02</td>
<td>-0.09</td>
<td>-0.09</td>
</tr>
<tr>
<td>Calc II</td>
<td>0.81</td>
<td>0.33</td>
<td>0.01</td>
<td>0.74</td>
<td>0.44</td>
</tr>
<tr>
<td></td>
<td>0.74</td>
<td>0.44</td>
<td>0.01</td>
<td>-0.07</td>
<td>-0.20</td>
</tr>
</tbody>
</table>

![Graph showing effect size](image-url)
“good teaching”

My Calculus Instructor:
• provided explanations that were understandable
• listened carefully to my questions and comments
• helped me become a better problem solver
• allowed time for me to understand difficult ideas
• made me feel comfortable in asking questions during class
• presented more than one method for solving problems
• made class interesting
• asked questions to determine if I understood what was being discussed
• discussed applications of calculus
• acted as if I was capable of understanding the key ideas of calculus
• encouraged students to seek help during office hours
• was available to make appointments outside of office hours, if needed
• did not make students feel nervous during class
• did not discourage me from wanting to continue taking Calculus
• encouraged students to enroll in Calculus II

How frequently did your instructor:
• prepare extra material to help students understand calculus concepts or procedures
• ask questions
• show how to work specific problems

Assignments completed outside of class time were: challenging but doable

My exams were graded fairly
My homework was graded fairly
My Calculus exams were a good assessment of what I learned
My primary role as a Calculus instructor is to

1. show students how to work problems
2. help students learn how to reason through problems
Calculus students learn best from lectures, provided they are clear and well-organized.
During class ...

- I was lost and unable to follow the lecture or discussion
- I simply copied whatever was written on the board

More than half of the students used at least half of their class time simply to copy whatever was written on the board.
Instructors were asked: “How frequently did you require students to explain their thinking on exams?”
Instructors were asked: “On a typical exam, what percentage of the points focused on skills and methods for carrying out computations?”

Mean: 54%
SD: 15%
Median: 50%
Q1: 40%
Q3: 70%
Marilyn Carlson and Michael Tallman, analysis of final exams
Adaptation of six intellectual behaviors from Anderson & Krathwohl (2001)

<table>
<thead>
<tr>
<th>Cognitive Behavior</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Remember</td>
<td>Students are prompted to retrieve knowledge from long-term memory (e.g., write the definition of the derivative).</td>
</tr>
<tr>
<td>Recall and apply procedure</td>
<td>Students must recognize what knowledge or procedures to recall when directly prompted to do so in the context of a problem (e.g., find the derivative/limit/integral of f).</td>
</tr>
<tr>
<td>Understand</td>
<td>Students are prompted to make interpretations, provide explanations, make comparisons or make inferences that require an understanding of a mathematics concept.</td>
</tr>
<tr>
<td>Apply understanding</td>
<td>Students must recognize when to use (or apply) a concept when responding to a question or when working a problem. To recognize the need to apply, execute or implement a concept in the context of working a problem requires an understanding of the concept.</td>
</tr>
<tr>
<td>Analyze</td>
<td>Students are prompted to break material into constituent parts and determine how parts relate to one another and to an overall structure or purpose. Differentiating, organizing, and attributing are characteristic cognitive processes at this level.</td>
</tr>
<tr>
<td>Evaluate</td>
<td>Students are prompted to make judgments based on criteria and standards. Checking and critiquing are characteristic cognitive processes at this level.</td>
</tr>
<tr>
<td>Create</td>
<td>Students are prompted to put elements together to form a coherent or functional whole; reorganize elements into a new pattern or structure. Generating, planning, and producing are characteristic cognitive processes at this level.</td>
</tr>
</tbody>
</table>
**Remember:** State the Mean Value Theorem.

**Recall and apply procedure:** Evaluate  \( \int_0^{\pi/4} \sin(x) \, dx \)

**Understand:** If \( r(x) \) represents the total revenue of company A from selling \( x \) units, interpret \( r'(4597) \).

**Apply understanding:** Find the value of \( x \) on the interval \([0, \sqrt{\pi}]\) that maximizes \( \sin(x^2) \).

**Analyze:** Write a one-page essay explaining why *limit* is a central theme of this course.
Distribution of the percentage of each exam that consisted of problems at each cognitive level.

By comparison, AP Calculus free response questions were 60.3% Recall and Apply, 39.7% Apply Understanding.
Distribution of the percentage of each exam that consisted of problems at each cognitive level.

Actual interquartile range: [70.5%, 88.2%]

Interquartile range based on instructor estimates: [40%, 70%]
Take away messages:

1. Students who arrive in Calculus I have high levels of interest in mathematics and a desire to understand it.

2. From the start to the end of the course, there is a large and significant decrease in student confidence in their mathematical abilities and enjoyment of mathematics.

3. The single greatest factor counteracting this trend that is under the control of the instructor is the quality of teaching as viewed by the students.
Take away messages:

4. Instructors tend to favor lecture format. Most students are not engaged by this format.

5. Assessment for most calculus classes is dominated by recalling and applying procedures, to an even greater extent than instructors are aware.

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