Characteristics of Successful Programs In College Calculus
A report on our findings

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PowerPoint available at www.macalester.edu/~bressoud/talks

Oregon State University
Corvallis, OR
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President's Council of Advisors on Science and Technology (PCAST)

February, 2012
Fall 2011: 240,000 students entered four-year undergraduate programs intending to major in engineering, a physical science, mathematics, or statistics.

An additional 300,000 are enrolled in one of these programs in a two-year college.

In 2010, we graduated 112,000 students with a Bachelor’s degree in one of these disciplines.
Hispanic Students as % of Total Degrees

<table>
<thead>
<tr>
<th>Year</th>
<th>% of total degrees</th>
<th>Engineering</th>
<th>Mathematics</th>
<th>Physical Sciences</th>
</tr>
</thead>
<tbody>
<tr>
<td>1990</td>
<td>2%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1992</td>
<td>3%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1994</td>
<td>4%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1996</td>
<td>5%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1998</td>
<td>6%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2000</td>
<td>7%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2002</td>
<td>8%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2004</td>
<td>9%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2006</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2008</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2010</td>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

NCES data
There were 1,089 Bachelors in Math or Stat earned by African-Americans in 1997. By 2010, that number was down to 854.
Math & Stats Majors by Gender

NCES data
MAA Study of Calculus

Characteristics of Successful Programs in College Calculus

PI: David Bressoud
co-PI’s:

Marilyn Carlson
ASU

Vilma Mesa
U Michigan

Michael Pearson
MAA

Chris Rasmussen
SDSU
Fall 2010
Phase I: Survey

1. Institutional questionnaire, chair
2. Calculus coordinator questionnaire
3. Instructor pre-term survey
4. Student 2\textsuperscript{nd} to 3\textsuperscript{rd} week survey
5. Student 2\textsuperscript{nd} to 3\textsuperscript{rd} last week survey
6. Instructor post-term survey
7. Collection of final exams and grades
Fall 2012
Phase II: Case Studies

Each of four teams will make a 2–3 day visit to four institutions, each team focusing on one type of institution:
  Two-year colleges
  Undergraduate colleges
  Comprehensive universities
  Research universities
Survey responses from

168 colleges and universities

660 instructors representing almost 900 Calculus I classes and over 34,000 students

14,000 students
61% of all Calculus I students took a calculus class in high school. 61% of them earned an A (37% of all Calc I students).

For 69% of those took Calc in HS, it was an AP Calculus course (42% of all Calc I students).

81% of the AP Calculus students took the AP exam (34% of all Calc I students).

60% of those who took the exam earned a 3 or higher (just over 20% of all Calc I students).
11.4% of all Calc I students had earned a 4 or higher on the AB exam or a 3 or higher on the BC exam.

0.9% earned a 5 on the BC Calculus exam.

1.7% took AB Calculus one year, BC Calculus the following year, and then took Calculus I when they got to college. Extrapolates to over 5,000 such students in Fall Calculus I each year.
These are good students:

Average SAT Math: 652, standard deviation = 76, Interquartile range [610,700]

95% believe they have knowledge and abilities to succeed in calculus

89% find using reasoning to solve math problems a satisfying experience

83% enjoy mathematics

65% would be taking this course even if it were not required
They want to understand calculus:

74% prefer to make sense of the mathematics rather than simply memorizing it

72% see the role of the instructor as helping students to reason through problems on their own rather than showing students how to work the problems

58% expect to earn an A in this course
Grade for college Calculus I:

- 22% A
- 28% B
- 23% C
- 27% D, F, or Withdrew
Dependent Variables

Pre- and post-test:

• I am confident in my mathematical abilities
• If I had a choice, I would continue to take mathematics
• I enjoy doing mathematics
• I intend to take Calc II

Post-test only:

• This course has increased my interest in taking more mathematics
Control Variables

Demographics
  Gender, SES, Race/Ethnicity

HS Math
  Math courses taken; if calculus, what kind; if AP exam, which and what score
  Grade in last HS math course
  SAT/ACT scores

College
  Prior college math, year in college, career intention
  Pre-survey value
Independent Variables

Student Level:

• Student beliefs and attitudes about learning mathematics
• Study habits
• Level of intellectual engagement with the course
• Experience with technology (graphing calculators and/or computer software)
Independent Variables

Classroom Level–instructor supplied:
- Class size
- Instructor experience and background
- Instructor beliefs, attitudes, and interests
- Assessment practices
- Out of class interactions with students
- Use of technology including use of web resources
- Textbook as well as additional instructional resources provided for students
Independent Variables

Classroom Level—student supplied:

- Student perceptions of instructional practices
- Use of technology
- Student perceptions of assessment practices
- The intellectual community outside of class
Independent Variables

Institutional Level:

• Placement procedures
• Technological support
• Institutional support for students
• Institutional support for instructors
55% men

97% full-time students

75% freshmen

76% White, 14% Asian, 5% Black, 10% Hispanic
Total college population: 73% White, 9% Asian, 12% Black, 12% Hispanic

75% intend to major in Science or Engineering
(Bio 30%, Eng 30%, Phys Sci 6%, CS 5%)
## Dependent Variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>SD</th>
<th>SE</th>
<th>Δ</th>
<th>ES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Confidence</td>
<td>3.89</td>
<td>1.01</td>
<td>0.02</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3.42</td>
<td>1.18</td>
<td>0.02</td>
<td>-0.47</td>
<td>-0.46</td>
</tr>
<tr>
<td>Enjoyment</td>
<td>3.63</td>
<td>1.27</td>
<td>0.02</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3.28</td>
<td>1.37</td>
<td>0.02</td>
<td>-0.35</td>
<td>-0.27</td>
</tr>
<tr>
<td>If I had a Choice...</td>
<td>1.93</td>
<td>1.02</td>
<td>0.02</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.84</td>
<td>1.08</td>
<td>0.02</td>
<td>-0.09</td>
<td>-0.09</td>
</tr>
<tr>
<td>Calc II</td>
<td>0.81</td>
<td>0.33</td>
<td>0.01</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.74</td>
<td>0.44</td>
<td>0.01</td>
<td>-0.07</td>
<td>-0.20</td>
</tr>
</tbody>
</table>

The diagram shows the effect size (change in units of SD of the pretest) for each variable.
“Good Teaching”

My Calculus Instructor:
• listened carefully to my questions and comments
• allowed time for me to understand difficult ideas
• presented more than one method for solving problems
• asked questions to determine if I understood what was being discussed
• discussed applications of calculus
• encouraged students to seek help during office hours
• frequently prepared extra material

Assignments were challenging but doable
My exams were graded fairly
My Calculus exams were a good assessment of what I learned
“Progressive Teaching”

My Calculus Instructor:
• Required me to explain my thinking on homework and exams
• Required students to work together
• Had students give presentations
• Held class discussions
• Put word problems in the homework and on the exams
• Put questions on the exams unlike those done in class
• Returned assignments with helpful feedback and comments
Interaction

Post-Survey Confidence

Low "progressive teaching"  High "progressive teaching"

- high "good teaching"
- low "good teaching"
My primary role as a Calculus instructor is to

1. Show students how to work problems
2. Help students learn how to reason through problems
3. (Low frequency)
4. (Low frequency)
5. (High frequency)
6. (High frequency)
Calculus students learn best from lectures, provided they are clear and well-organized.
During class ...

- 45%: I was lost and unable to follow the lecture or discussion.
- 30%: I simply copied whatever was written on the board.

More than half of the students used at least half of their class time simply to copy whatever was written on the board.
Students who enter intending a STEM major and continuing to Calculus II:

12.5% have changed their mind about continuing to Calculus II by end of course.

81% of them received a C or higher for Calculus I

17% of women switched out

9% of men switched out

17.5% of those at large research universities (> 20,000 students) switched out
Analysis of “switchers”, those who originally intended to take Calculus II and pursue STEM major but changed their mind:

No significant differences attributable to

- Ethnic or racial status
- SAT/ACT scores
- Studying calculus in high school
- Amount of time spent working
- Amount of time spent studying
Analysis of “switchers”, those who originally intended to take Calculus II and pursue STEM major but changed their mind:

Switchers are significantly more likely to see success in Calculus as dependent on ability to solve specific types of problems. \( p < 0.001 \)

Non-switchers see success as a matter of making connections and forming logical arguments.
Analysis of “switchers”, those who originally intended to take Calculus II and pursue STEM major but changed their mind:

Switchers were far more likely to report that instructor did not engage them during class time.

Switchers were far more likely to report that they did not feel supported or encouraged by their instructor.

Switchers reported that calculus instruction was ineffective and uninspiring, course was “over stuffed” with content, and pace was too fast.
Take away messages:

1. Students who arrive in Calculus I have high levels of interest in mathematics and a desire to understand it.

2. From the start to the end of the course, there is a large and significant decrease in student confidence in their mathematical abilities and enjoyment of mathematics.

3. The single greatest factor counteracting this trend that is under the control of the instructor is the quality of teaching as viewed by the students.

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Marilyn Carlson and Michael Tallman, analysis of final exams
Adaptation of six intellectual behaviors from Anderson & Krathwohl (2001)

<table>
<thead>
<tr>
<th>Cognitive Behavior</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Remember</td>
<td>Students are prompted to retrieve knowledge from long-term memory (e.g., write the definition of the derivative).</td>
</tr>
<tr>
<td>Recall and apply procedure</td>
<td>Students must recognize what knowledge or procedures to recall when directly prompted to do so in the context of a problem (e.g., find the derivative/limit/integral of f).</td>
</tr>
<tr>
<td>Understand</td>
<td>Students are prompted to make interpretations, provide explanations, make comparisons or make inferences that require an understanding of a mathematics concept.</td>
</tr>
<tr>
<td>Apply understanding</td>
<td>Students must recognize when to use (or apply) a concept when responding to a question or when working a problem. To recognize the need to apply, execute or implement a concept in the context of working a problem requires an understanding of the concept.</td>
</tr>
<tr>
<td>Analyze</td>
<td>Students are prompted to break material into constituent parts and determine how parts relate to one another and to an overall structure or purpose. Differentiating, organizing, and attributing are characteristic cognitive processes at this level.</td>
</tr>
<tr>
<td>Evaluate</td>
<td>Students are prompted to make judgments based on criteria and standards. Checking and critiquing are characteristic cognitive processes at this level.</td>
</tr>
<tr>
<td>Create</td>
<td>Students are prompted to put elements together to form a coherent or functional whole; reorganize elements into a new pattern or structure. Generating, planning, and producing are characteristic cognitive processes at this level.</td>
</tr>
</tbody>
</table>
Remember: State the Mean Value Theorem.

Recall and apply procedure: Evaluate \( \int_0^{\pi/4} \sin(x) \, dx \)

Understand: If \( r(x) \) represents the total revenue of company A from selling \( x \) units, interpret \( r'(4597) \).

Apply understanding: Find the value of \( x \) on the interval \([0, \sqrt{\pi}]\) that maximizes \( \sin x^2 \).

Analyze: Write a one-page essay explaining why \( \text{limit} \) is a central theme of this course.
Distribution of the percentage of each exam that consisted of problems at each cognitive level.

By comparison, AP Calculus free response questions were 60.3% Recall and Apply, 39.7% Apply Understanding.
Distribution of the percentage of each exam that consisted of problems at each cognitive level.

Actual interquartile range: [70.5%, 88.2%]

Interquartile range based on instructor estimates: [40%, 70%]