Characteristics of Successful Programs In College Calculus
A report on our findings

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PowerPoint available at
www.macalester.edu/~bressoud/talks
MAA Study of Calculus

Characteristics of Successful Programs in College Calculus

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NSF #0910240
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Phase I: Survey

1. Institutional questionnaire, chair
2. Calculus coordinator questionnaire
3. Instructor pre-term survey
4. Student 2\textsuperscript{nd} to 3\textsuperscript{rd} week survey
5. Student 2\textsuperscript{nd} to 3\textsuperscript{rd} last week survey
6. Instructor post-term survey
7. Collection of final exams and grades
Initial survey responses from
168 colleges and universities
660 instructors representing almost 900 Calculus I classes and over 34,000 students
14,000 students
61% all Calculus I students took a calculus class in high school. 61% of them earned an A (37% of all Calc I students)

For 69% of those took Calc in HS, it was an AP Calculus course (42% of all Calc I students).

81% of the AP Calculus students took the AP exam (34% of all Calc I students)

60% of those who took the exam earned a 3 or higher (just over 20% of all Calc I students)
These are good students:

Average SAT Math: 652, standard deviation = 76, Interquartile range [610,700]

95% believe they have knowledge and abilities to succeed in calculus

89% find using reasoning to solve math problems a satisfying experience

83% enjoy mathematics

65% would be taking this course even if it were not required
They want to understand calculus:

74% prefer to make sense of the mathematics rather than simply memorizing it

72% see the role of the instructor as helping students to reason through problems on their own rather than showing students how to work the problems

58% expect to earn an A in this course
Grade for college Calculus I:

22% A

28% B

23% C

27% D, F, or Withdrew
My primary role as a Calculus instructor is to

1. show students how to work problems
2. help students learn how to reason through problems
3.
4.
5.
6. help students learn how to reason through problems
Calculus students learn best from lectures, provided they are clear and well-organized.
During class ...

- 45% never
- 40% sometimes about half the time
- 30% most of the time
- 25% every class

- 35% I was lost and unable to follow the lecture or discussion
- 20% I simply copied whatever was written on the board

More than half of the students used at least half of their class time simply to copy whatever was written on the board.
Comparison of start of term and end of term surveys

Statistical analysis by

Phil Sadler  Gerhard Sonnert
Harvard University
Dependent Variables

Pre- and post-term surveys:

• I am confident in my mathematical abilities
• If I had a choice, I would continue to take mathematics
• I enjoy doing mathematics
• I intend to take Calc II

Post-term survey only:

• This course has increased my interest in taking more mathematics
Selection Bias

While the grade distribution of all students (as reported by instructors) was
A: 22%, B: 28%, C: 23%, DFW: 27%

For those who completed both surveys, it was
A: 38%, B: 39%, C: 19%, DFW: 4%
Control Variables

Demographics
  Gender, SES, Race/Ethnicity

HS Math
  Math courses taken; if calculus, what kind; if AP exam, which and what score
  Grade in last HS math course
  SAT/ACT scores

College
  Prior college math, year in college, career intention
  Pre-term survey value
Independent Variables

Student supplied:

- Student beliefs and attitudes about learning mathematics
- Study habits
- Level of intellectual engagement with the course
- Experience with technology (graphing calculators and/or computer software)
Independent Variables

Student supplied:

• Student perceptions of instructional practices
• Student perceptions of instructor use of technology
• Student perceptions of assessment practices
• The intellectual community outside of class
Independent Variables

Instructor supplied:
• Class size
• Instructor experience and background
• Instructor beliefs, attitudes, and interests
• Assessment practices
• Out of class interactions with students
• Use of technology including use of web resources
• Textbook as well as additional instructional resources provided for students
Independent Variables

Coordinator supplied:
• Placement procedures
• Technological support
• Institutional support for students
• Institutional support for instructors
### Dependent Variables

<table>
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<th>Variable</th>
<th>Mean</th>
<th>SD</th>
<th>SE</th>
<th>Δ</th>
<th>ES</th>
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<td>1.18</td>
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</table>
Pre-term survey value was the greatest single predictor of post-survey value.

Science majors did better, effect sizes of 
0.52*** on increased interest in math 
0.28*** on enjoyment 
0.25*** on desire to take more math 
0.19* on confidence

* $p < 0.05$, ***$p < 0.001$
Being male had a large effect size:
0.60*** on continuing to Calculus II
0.39*** on increased interest in math
0.17*** on confidence
0.11* on desire to take more math

Taking AP Calculus had a large effect size:
0.31*** on confidence if AB Calculus
0.41*** on confidence if BC Calculus

* $p < 0.05$, ***$p < 0.001$
“Good Teaching”

My Calculus Instructor:
• listened carefully to my questions and comments
• allowed time for me to understand difficult ideas
• presented more than one method for solving problems
• asked questions to determine if I understood what was being discussed
• discussed applications of calculus
• encouraged students to seek help during office hours
• frequently prepared extra material

Assignments were challenging but doable
My Calculus exams were a good assessment of what I learned

\[ p < 0.001 \]
“Progressive Teaching”

My Calculus Instructor:
• Required me to explain my thinking on homework and exams
• Required students to work together
• Had students give presentations
• Held class discussions
• Put word problems in the homework and on the exams
• Put questions on the exams unlike those done in class
• Returned assignments with helpful feedback and comments

Significant interaction with good teaching
Interaction

Low "progressive teaching"  High "progressive teaching"

Post-Survey Confidence

0 1 2 3 4 5

high "good teaching"

low "good teaching"
Switcher Analysis

Chris Rasmussen
Jess Ellis
Kristin Duncan
Students who enter intending a STEM major and continuing to Calculus II:

12.5% have changed their mind about continuing to Calculus II by end of course.

17% of women switched out
9% of men switched out

17.5% of those at large research universities (> 20,000 students) switched out
Analysis of “switchers”, those who originally intended to take Calculus II and pursue STEM major but changed their mind:

No significant differences attributable to

- Ethnic or racial status
- SAT/ACT scores
- Studying calculus in high school
- Amount of time spent working
- Amount of time spent studying
Analysis of “switchers”, those who originally intended to take Calculus II and pursue STEM major but changed their mind:

Switchers are significantly more likely to see success in Calculus as dependent on ability to solve specific types of problems. ($p < 0.001$)

Non-switchers see success as a matter of making connections and forming logical arguments.
Analysis of “switchers”, those who originally intended to take Calculus II and pursue STEM major but changed their mind:

Switchers were far more likely to report that instructor did not engage them during class time.

Switchers were far more likely to report that they did not feel supported or encouraged by their instructor.

Switchers reported that calculus instruction was ineffective and uninspiring, course was “over stuffed” with content, and pace was too fast.
Remember: State the Mean Value Theorem.

Recall and apply procedure: Evaluate $\int_0^{\pi/4} \sin(x)\,dx$

Understand: If $r(x)$ represents the total revenue of company A from selling $x$ units, interpret $r’(4597)$.

Apply understanding: Find the value of $x$ on the interval $[0, \sqrt{\pi}]$ that maximizes $\sin x^2$.

Analyze: Write a one-page essay explaining why limit is a central theme of this course.
Distribution of the percentage of each exam that consisted of problems at each cognitive level.

Actual interquartile range: [70.5%, 88.2%]

Interquartile range based on instructor estimates: [40%, 70%]
Take away messages:

1. Students who arrive in Calculus I have high levels of interest in mathematics and a desire to understand it.

2. From the start to the end of the course, there is a large and significant decrease in student confidence in their mathematical abilities and enjoyment of mathematics. This is especially pronounced for women and those who have not studied calculus in high school,

3. The single greatest factor counteracting this trend that is under the control of the instructor is the quality of teaching as viewed by the students.
Take away messages:

4. Instructors tend to favor lecture format. Most students are not engaged by this format. Students are most likely to continue with mathematics if they find themselves making connections and needing to reason through the concepts.

5. Assessment for most calculus classes is dominated by recalling and applying procedures, to an even greater extent than instructors are aware.

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