Calculus in High School
Too much of a good thing?

David Bressoud
Macalester College
Saint Paul, MN

Park City, Utah
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PowerPoint available at
www.macalester.edu/~bressoud/talks
In the Fall of 2011: 240,000 students entered four-year undergraduate programs with the intention of majoring in engineering, a physical science, mathematics, or statistics.

Less than half will succeed.
Students in college or university Calculus I:

Mean score on SAT Math: 652
75% earned 610 or higher (top 23%)

Mean score on ACT Math: 28.5
75% earned 26 or higher (top 16%)

MAA survey of 700 instructors, over 14,000 students, all types of colleges and universities across US, Fall 2010
61% all Calculus I students took a calculus class in high school. 61% of them earned an A (37% of all Calc I students)

For 69% of those took Calc in HS, it was an AP Calculus course (42% of all Calc I students).

81% of the AP Calculus students took the AP exam (34% of all Calc I students)

60% of those who took the exam earned a 3 or higher (just over 20% of all Calc I students)

MAA survey of 700 instructors, over 14,000 students, all types of colleges and universities across US, Fall 2010
Grade for college Calculus I:

- 22% A
- 28% B
- 23% C
- 27% D, F, or Withdrew
Me and AP:

1968 took AP Calculus exam

1990–1991 taught AP Calculus at State College Area High School

1993–2007 AP Reading
   (Reader, Table Leader, Question Leader)

1999–2005 AP Calculus Development Committee
   (Chair from 2002 to 2005)
The Rocky Transition From High-School Calculus

http://chronicle.com/article/High-School-Calculus-The-E/63533/
Fall Enrollments in Calculus I versus AP Calculus Exams (thousands)

Over 600,000 students are studying calculus in high school this year, over 1/3 of the 1.8 million who will go directly from HS to college.

CBMS and College Board data
Math-intensive Bachelor's degrees relative to number of 22 year-olds (degrees ÷ # of 22-year olds)

- Engineering
- Physical Science
- Math & Stat

NCES & US Census data
A quick History of AP Calculus
1953–55:
College Admission with Advanced Standing

Bowdoin, Brown, Carleton, Haverford, Kenyon, MIT, Middlebury, Oberlin, Swarthmore, Wabash, Wesleyan, and Williams

1956:
First Advanced Placement exams administered by College Board
1970’s and 1980’s

Jaime Escalante

1984: South Carolina’s Education Improvement Act

Richard Riley, Governor of South Carolina at the time, later Secretary of Education under President Clinton
1990’s

1995: Graphing Calculators

1998: AB subscore

Today, two-thirds of the exam is calculator-free, one-third allows and may require use of graphing calculator.

Students who do best on both parts of exam have teachers who allow use of calculators $\frac{1}{4}$ to $\frac{1}{2}$ of time.
1997–98 exams based on new syllabus

• Graphical, numerical, analytical, and verbal descriptions of functions

Note: This is the graph of the derivative of $f$, not the graph of $f$.

The figure above shows the graph of $f'$, the derivative of a function $f$. The domain of $f$ is the set of all real numbers $x$ such that $-10 \leq x \leq 10$.

(a) For what values of $x$ does the graph of $f$ have a horizontal tangent?
(b) For what values of $x$ in the interval $(-10, 10)$ does $f$ have a relative maximum? Justify your answer.
(c) For what values of $x$ is the graph of $f$ concave downward?
A particle moves along the $y$-axis so that its velocity $v$ at time $t \geq 0$ is given by $v(t) = 1 - \tan^{-1}(e^t)$. At time $t = 0$, the particle is at $y = -1$. Find the position of the particle at time $t = 2$.

\[
y(x) = \int_0^x v(t) \, dt + C
\]

\[
y(0) = \int_0^0 v(t) \, dx + C = C
\]

\[
\Rightarrow C = -1
\]

\[
y(2) = \int_0^2 \left(1 - \tan^{-1}(e^t)\right) \, dt - 1 = -1.36069
\]
2004 AB3(d)

A particle moves along the y-axis so that its velocity \( v \) at time \( t \geq 0 \) is given by \( v(t) = 1 - \tan^{-1}(e^{t}) \). At time \( t = 0 \), the particle is at \( y = -1 \). Find the position of the particle at time \( t = 2 \).

\[
\int_{0}^{2} v(t) \, dt = y(2) - y(0)
\]

\[
y(2) = y(0) + \int_{0}^{2} \left(1 - \tan^{-1}(e^{t})\right) \, dt
\]

\[
= -1 + \int_{0}^{2} \left(1 - \tan^{-1}(e^{t})\right) \, dt = -1.36069
\]
1997–98 exams based on new syllabus

• Be able to communicate mathematics: justify local or absolute extremum, explain the meaning of an answer

2005 AB5/BC5

A car is traveling on a straight road. For $0 \leq t \leq 24$ seconds, the car’s velocity $v(t)$, in meters per second, is modeled by the piecewise-linear function defined by the graph above.

(a) Find $\int_{0}^{24} v(t) \, dt$. Using correct units, explain the meaning of $\int_{0}^{24} v(t) \, dt$.

(b) For each of $v'(1)$ and $v'(20)$, find the value or explain why it does not exist. Indicate units of measure.

(c) Let $a(t)$ be the car’s acceleration at time $t$, in meters per second per second. For $0 < t < 24$, write a piecewise-defined function for $a(t)$.

(d) Find the average rate of change of $v$ over the interval $8 \leq t \leq 20$. Does the Mean Value Theorem guarantee a value of $c$, for $8 < c < 20$, such that $v'(c)$ is equal to this average rate of change? Why or why not?
There were 1,089 Bachelors in Math or Stat earned by African-Americans in 1997. By 2010, that number was down to 854.
How effective is AP Calculus?
Performance of students who used AP credit to go straight into Calculus II

<table>
<thead>
<tr>
<th>AB score</th>
<th>Average Grade in Calc II*</th>
<th>BC score</th>
<th>Average Grade in Calc II*</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>C+/B-</td>
<td>3</td>
<td>B</td>
</tr>
<tr>
<td>4</td>
<td>B-/B</td>
<td>4</td>
<td>B+</td>
</tr>
<tr>
<td>5</td>
<td>B+</td>
<td>5</td>
<td>B+</td>
</tr>
</tbody>
</table>

*grades weighted so that SAT scores are comparable to those of students who had not skipped Calculus I

Morgan & Klaric (1994) study of 22 colleges and universities**
Keng & Dodd (2001) study at U of Texas, Austin

Students who earned 3 or higher on AB exam and chose to retake Calculus I did worse in Calculus II then those who went directly to Calculus II.

_Caveats:_ Difference was statistically significant at .05 only 1 out of 4 years.
Not controlled for comparability of ability levels
Phil Sadler

Factors Influencing College Success in Mathematics

There is some benefit from simply taking a calculus class in high school:
non-AP, AP with no exam, AP and earn < 3 on exam each raised expected grade in Calculus I by 1 or 2 points (out of 100)
There was no statistically significant benefit attributable to any non-calculus courses, including AP Statistics.
Taking AP Calculus and earning a 3 or higher on AB or BC exam did provide a benefit of 3 to 10 points in Calculus I.
Phil Sadler

Factors Influencing College Success in Science

Students who study Calculus in HS and do well on AP exam ($\geq 3$ on AB exam) also do significantly better in intro Biology, Chemistry, and Physics.
Outcome of Calculus in HS (thousands)

- Never earn credit for college math above precalc, bottom 100,000 take remedial math
- DE no Calc I, other
- Retake Calc I, little or no benefit from HS calc
- AP ≥ 3, Retake Calc I, some benefit
- AP ≥ 3, use credit for Calc I

All numbers are rough approximations

≥ 3 on AP Exam

Take Calc I in college
MAA/NCTM Joint Position on Calculus
(adopted March 2012)

1. Students who enroll in a calculus course in secondary school should have demonstrated mastery of algebra, geometry, trigonometry, and coordinate geometry;
MAA/NCTM Joint Position on Calculus  
(adopted March 2012)

2. The calculus course offered in secondary school should have the substance of a mainstream college-level course;
MAA/NCTM Joint Position on Calculus
(adopted March 2012)

3. The college curriculum should acknowledge the ubiquity of calculus in secondary school, shape the calculus curriculum so that it is appropriate for those who have experienced introductory calculus, and offer alternatives to calculus.

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