Calculus in High School

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This PowerPoint will be available at
www.macalester.edu/~bressoud/talks
High school graduates, 2007: $3,232,000$

2006 high school graduates who entered college within a year: $1,850,000$

2004 Bachelor’s degrees: $1,400,000$

- engineering & eng. technology: 78,200
- biological sciences: 61,500
- computer & information science: 59,500
- physical sciences: 18,000
- math & stat: 13,300

$230,500$

Dept. of Ed, NCES data
<table>
<thead>
<tr>
<th>year</th>
<th>% majoring in math</th>
</tr>
</thead>
<tbody>
<tr>
<td>1975</td>
<td>2.03%</td>
</tr>
<tr>
<td>1980</td>
<td>1.23%</td>
</tr>
<tr>
<td>1985</td>
<td>1.33%</td>
</tr>
<tr>
<td>1990</td>
<td>1.22%</td>
</tr>
<tr>
<td>1995</td>
<td>1.07%</td>
</tr>
<tr>
<td>2000</td>
<td>0.87%</td>
</tr>
<tr>
<td>2005</td>
<td>0.86%</td>
</tr>
</tbody>
</table>

CBMS data
Mainstream Calculus I Enrollments
(fall only for 2- & 4-yr colleges & universities)

2007: 276,004

CBMS and College Board data
AP Calculus currently growing at >14,000/year (about 6%)
AP Calculus currently growing at >14,000/year (about 6%)

Estimated # of students taking Calculus in high school (NAEP, 2005): ~ 500,000

Estimated # of students taking Calculus I in college: ~ 500,000 (includes Business Calc)
High School Calculus breakdown:
380,000 take AP Calculus (College Board estimate)
50,000 take IB or Dual Enrollment
100,000 take another course with “Calculus” in the title
High School Calculus breakdown:

380,000 take AP Calculus (College Board estimate)

276,004 took AP exam; AB: 211,693, BC: 64,311
High School Calculus breakdown (2007 numbers):
380,000 take AP Calculus (College Board estimate)
276,004 took AP exam; AB: 211,693, BC: 64,311

<table>
<thead>
<tr>
<th>Test</th>
<th>Grade</th>
<th>Credit-AP</th>
<th>Grade</th>
<th>Credit-AP</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB</td>
<td>5</td>
<td>84%-77%</td>
<td>BC</td>
<td>79%-73%</td>
</tr>
<tr>
<td>AB</td>
<td>4</td>
<td>82%-64%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AB</td>
<td>3</td>
<td>60%-39%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AB</td>
<td>1-2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BC</td>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

Percentages from 2002 nationwide random sample for *The College Board* by Karen Morgan, N = 435 (AB), = 139 (BC)
Based on the estimates by Morgan, of 276,000 who took the AP exam in 2007, roughly 145,000 receive college credit for calculus, roughly 125,000 of them use this credit to go directly to the next math class.
Fall of 1991

Boston College, Brigham Young University, Carnegie Mellon University, Clemson University, College of William and Mary, Cornell College (IA), Cornell University, Duke University, Michigan State University, Pennsylvania State University, Stanford University, Tulane University, UC-Davis, UC-Irvine, University of Georgia, University of Illinois, UNC-Chapel Hill, UT-Austin, University of Utah, University of Virginia, Yale University

Average grade in Calculus II:

Took Calc I at that university: 2.52
3 on AB exam and used AP: 2.67
4 on AB exam and used AP: 2.79
5 on AB exam and used AP: 3.23

Morgan & Ramist, 1998
Fall of 1991

Boston College, Brigham Young University, Carnegie Mellon University, 
Clemson University, College of William and Mary, Cornell College (IA), 
Cornell University, Duke University, Michigan State University, Pennsylvania 
State University, Stanford University, Tulane University, UC-Davis, UC-Irvine, 
University of Georgia, University of Illinois, UNC-Chapel Hill, UT-Austin, 
University of Utah, University of Virginia, Yale University

Average grade in Calculus III:

Took Calc II at that university: 2.87

3 on BC exam and used AP: 3.11

4 on BC exam and used AP: 2.74

5 on BC exam and used AP: 3.32
1996–99

University of Texas at Austin

Average grade in Calculus II:

Took Calc I at that university: 2.55
3–5 on AB exam and used AP: 2.98

Non-AP students chosen so that SAT scores are comparable for both sets of students

Dodd et al, 2002
Fall of 1994


Average grade in Calculus II:  
SAT-adjusted

Took Calc I at that university: 2.43

3 on AB exam and used AP: 2.69  2.64

4 on AB exam and used AP: 2.90  2.78

5 on AB exam and used AP: 3.34  3.15

Morgan & Klaric, 2006
Fall of 1994

Barnard College, Binghamton U., Brigham Young U., Carnegie Mellon U.,
College of William and Mary, Cornell U., Dartmouth College, George
Washington U., GA Institute of Technology, Miami U. (OH), NC State U.,
Northwestern U., Stanford U., Texas A&M, UC-Davis, UC-LA, U. of FL, U. of
IL-Urbana, U. of IA, U. of MD, U. of Miami, USC, UT-Austin, U. of VA, U. of
WA, Wesleyan College, Williams College

<table>
<thead>
<tr>
<th>Average grade in Calculus III:</th>
<th>SAT-adjusted</th>
</tr>
</thead>
<tbody>
<tr>
<td>Took Calc II at that university:</td>
<td>2.50</td>
</tr>
<tr>
<td>3 on BC exam and used AP:</td>
<td>3.00</td>
</tr>
<tr>
<td></td>
<td>2.92</td>
</tr>
<tr>
<td>4 on BC exam and used AP:</td>
<td>3.45</td>
</tr>
<tr>
<td></td>
<td>3.35</td>
</tr>
<tr>
<td>5 on BC exam and used AP:</td>
<td>3.46</td>
</tr>
<tr>
<td></td>
<td>3.27</td>
</tr>
</tbody>
</table>

Morgan & Klaric, 2006
Classes of 1990 & 1991
College of William & Mary

Percentage taking next math class (SAT-adjusted)
≥ 4 on AB or ≥ 3 on BC: 60%

Took Calculus I at W&M: 61%

Percentage that eventually majored in Mathematics (SAT-adjusted)
≥ 4 on AB or ≥ 3 on BC: 3%

Took Calculus I at W&M: 8%

David Lutzer, private communication
Conclusion:

Students who take AP Calculus in high school and do well on the exam generally are as well prepared for the next class as those coming out of the comparable college courses.

But, this is a very large group (about 145,000) of very talented students. Are the colleges and universities doing all they can and should to entice these students to continue mathematics and to prepare them for mathematically intensive majors?
Mainstream Calculus II Enrollments (fall only for 2- & 4- year colleges and

students (thousands)

academic year

2007: 64,311

4-yr colleges & universities
2-yr colleges
AP Calculus BC only

CBMS & College Board data
Calculus Before Grade 12

College Board data
BC exam,
8,818 in 2002
15,533 in 2007
76% increase

College Board data
American Math Competition survey, spring 2007, 3516 high schools

1582 reported at least some students take highest level of calculus offered before senior year.

1114 reported that at least some of these students take no mathematics in their senior year. In 203 of these schools, more than 50% of these students take no math in their senior year.

Reasons:

• Taking AP courses in other subjects - 580
• Intended major doesn’t require any more math - 458
• No courses available - 245
Conclusion:

Students completing calculus before their senior year are the best of the best, and their numbers are significant. What can colleges and universities do to encourage them to continue their study of mathematics?
Roughly 130,000 took AP exam, did not earn or chose not to accept credit. Morgan data suggests that about 2/3rds plan on retaking calculus in college.

Those who take calculus in high school but not receive college credit for it consist of:

~100,000 take generic “calculus”;
~105,000 take AP calculus but not the exam;
105,000 take AP exam and score less than 3;
~30,000 score 3 or better on AP exam but do not receive credit or elect not to use it.
About 340,000 students arrived at college this fall having taken calculus in high school but without bringing college credit for it.
US Dept. of Ed. transcript study, high school class of 1992, 30% of those who took a course called “calculus” in high school took precalculus when they got to college. If extrapolated to current numbers, that would be over 150,000.
Conclusion:

Colleges and universities need to pay serious attention to these students. They constitute a very large proportion of those who seek mathematically-intensive majors.

Although we need more data on what happens to these students, simply treating them as though they have not seen calculus before does not work.
College enrollment in Calculus I has been constant at about half a million students per year over the past quarter century.

During this time, the number of students arriving with credit for Calculus I has gone from a negligible number to about $145,000 + 30,000 = 175,000$.

The top third of the Calculus I students have disappeared and been replaced by an equal number of students who would not have taken Calculus in 1982.

Probably about half of all students taking Calculus I in college took a calculus class in high school.
Conclusion:

If students in college Calculus I seem less well prepared today than they did 20 years ago, it is because we are looking at a different segment of the student population.

This is not a reason to lower expectations, but it is reason to reconsider what it is we really want these students to learn and how we can adjust our pedagogy to help them accomplish it.
At 5-year intervals starting in 1990, CBMS has been tracking number of sections of mainstream Calculus I that use various markers of reform calculus:

• Use of graphing calculators
• Use of computer assignments
• Use of writing assignments
• Use of group projects

Results from 2005 are just in.
Group Projects

CBMS data
Use of online resources in mainstream Calculus I (2005):

PhD: 9%

MA: 2%

BA: 2%

2-year: 5%
Conclusion:

Reform fatigue is a serious issue. Those who have found reforms that work need to generate broad buy-in within their departments. This includes finding mechanisms (such as on-line resources) that assist faculty to fully engage students.
Mainstream Calculus II, fall

students (thousands)

year


TY 80 100 120
BA 70 90 110
MA 60 80 100
PhD 50 70 90

CBMS data
Who teaches Calc II at PhD institutions

- Graduate TA
- Part-time
- Other full-time
- T/TE

CBMS data
Conclusion:

A lot of the responsibility for the future of all STEM majors rests with our research universities. Research mathematicians cannot afford to ignore what is happening to first-year students in their university.
Dual enrollment

In spring, fall 2005 (combined), 33,436 students studied Calculus I under dual enrollment programs: 14,030 in connection with 4-year colleges, 19,406 in connection with 2-year colleges.

<table>
<thead>
<tr>
<th>Control of</th>
<th>4-year colleges</th>
<th>2-year colleges</th>
</tr>
</thead>
<tbody>
<tr>
<td>syllabus</td>
<td>92%</td>
<td>89%</td>
</tr>
<tr>
<td>textbook</td>
<td>44%</td>
<td>74%</td>
</tr>
<tr>
<td>instructor</td>
<td>48%</td>
<td>52%</td>
</tr>
<tr>
<td>final exam</td>
<td>30%</td>
<td>37%</td>
</tr>
</tbody>
</table>
Phil Cheifetz, Nassau Community College

*The NCC Partnership Program in Mathematics*

- Training of high school teachers (one-week workshop plus regularly scheduled users’ group)
- Close supervision, college supervisor teaches the class once every two weeks
- College writes *and grades* the exams.
Early College High School

Goal: To give young people from groups underrepresented in higher education an opportunity to study college-level material while in high school, and to earn up to two full years of college credit while in high school.

Spearheaded by the Bill & Melinda Gates Foundation with support from Carnegie Corporation of New York, The Ford Foundation, and The W.K. Kellogg Foundation, it is a serious undertaking that is already operating in 130 schools, involving over 16,000 students. The goal by 2012 is 239 schools and 96,400 students.

For more, see my July Launchings: “The Dangers of Dual Enrollment” http://www.maa.org/columns/launchings/launchings_07_07.html
Conclusion:

Dual enrollment should be a concern of all of us. It can be done well or it can be done poorly. Without vigilance and accountability, there is a very real danger that it will done poorly and on a very large scale.
Additional Conclusions:

1. We need more current and comprehensive data about what happens to students as they cross the divide from high school to college.

2. We need to rethink calculus instruction and how to meet the needs of the constituencies that did not exist 25 years ago:
   - Those who pass out of calculus;
   - Those who are retaking a course they took in high school;
   - Those who studied calculus in high school but are not deemed ready for it when they get to college;
   - Those who would not have taken calculus 25 years ago.
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