Calculus as a High School Course

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Barrett Lectures: Transitions To, Through, and Beyond Calculus

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This PowerPoint will be available at
www.macalester.edu/~bressoud/talks
More information and detailed references in my monthly
Launchings columns at www.maa.org
<table>
<thead>
<tr>
<th>year</th>
<th>% majoring in math</th>
</tr>
</thead>
<tbody>
<tr>
<td>1975</td>
<td>2.03%</td>
</tr>
<tr>
<td>1980</td>
<td>1.23%</td>
</tr>
<tr>
<td>1985</td>
<td>1.33%</td>
</tr>
<tr>
<td>1990</td>
<td>1.22%</td>
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<tr>
<td>1995</td>
<td>1.07%</td>
</tr>
<tr>
<td>2000</td>
<td>0.87%</td>
</tr>
<tr>
<td>2005</td>
<td>0.86%</td>
</tr>
</tbody>
</table>

CBMS data
At Macalester College, following MAA’s *CUPM Curriculum Guide 2004*, we require for the math major:

• **Discrete Math** (with emphasis on problem solving)

• **Computer Science** (may be satisfied by first course for the Computer Science major or by course in Scientific Computing)

• A challenging and interesting introduction to **Statistics** (with emphasis on statistical modeling)

These provide alternative courses for first-year students that are entries to the major.
High school graduates, 2007: 3,232,000

2006 high school graduates who entered college within a year: 1,850,000

2004 Bachelor’s degrees: 1,400,000

- engineering & eng. technology: 78,200
- biological sciences: 61,500
- computer & information science: 59,500
- physical sciences: 18,000
- math & stat: 13,300

230,500

Dept. of Ed, NCES data
Mainstream Calculus I Enrollments
(fall only for 2- & 4-yr colleges & universities)

- 4-yr colleges & universities
- 2-yr colleges
- AP Calculus (AB & BC)

CBMS and College Board data
AP Calculus has been growing at ~17,000/year (about 7%)
Estimated # of students taking Calculus in high school (NAEP, 2005): ~ 500,000

Estimated # of students taking Calculus I in college: ~ 500,000 (includes Business Calc)

AP Calculus has been growing at ~17,000/year (about 7%)
High School Calculus breakdown:

380,000 take AP Calculus (College Board estimate)

276,004 took AP exam; AB: 211,693, BC: 64,311

50,000 take IB or Dual Enrollment

100,000 take another course with “Calculus” in title
80–85% of those who earn 4 or 5 on AB exam and almost all who earn 3+ on BC receive at least some college credit. For 3+ on BC exam and for those with 5 on AB, they usually also take at least some Advanced Placement.

<table>
<thead>
<tr>
<th>AB exam:</th>
<th>credit-AP</th>
<th>BC exam:</th>
<th>79%-73%</th>
</tr>
</thead>
<tbody>
<tr>
<td>5: 44,500</td>
<td>84%-77%</td>
<td>28,000</td>
<td></td>
</tr>
<tr>
<td>4: 39,600</td>
<td>82%-64%</td>
<td>11,500</td>
<td></td>
</tr>
<tr>
<td>3: 40,400</td>
<td>60%-39%</td>
<td>12,100</td>
<td></td>
</tr>
<tr>
<td>1–2: 92,500</td>
<td></td>
<td>12,700</td>
<td></td>
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</tbody>
</table>

Percentages from 2002 nationwide random sample for *The College Board* by Karen Morgan, N = 435 (AB), = 139 (BC)
Comparison of AB exam vs Calc I

Fall of 1991

Boston College, Brigham Young University, Carnegie Mellon University, Carnegie Mellon University, College of William and Mary, Cornell College (IA), Cornell University, Duke University, Michigan State University, Pennsylvania State University, Stanford University, Tulane University, UC-Davis, UC-Irvine, University of Georgia, University of Illinois, UNC-Chapel Hill, UT-Austin, University of Utah, University of Virginia, Yale University

Average grade in Calculus II:

Took Calc I at that university: 2.52
3 on AB exam and used AP: 2.67
4 on AB exam and used AP: 2.79
5 on AB exam and used AP: 3.23
Fall of 1991

Comparison of BC exam vs Calc II

Boston College, Brigham Young University, Carnegie Mellon University, Carnegie Mellon University, College of William and Mary, Cornell College (IA), Cornell University, Duke University, Michigan State University, Pennsylvania State University, Stanford University, Tulane University, UC-Davis, UC-Irvine, University of Georgia, University of Illinois, UNC-Chapel Hill, UT-Austin, University of Utah, University of Virginia, Yale University

Average grade in Calculus III:

Took Calc II at that university: 2.87
3 on BC exam and used AP: 3.11
4 on BC exam and used AP: 2.74
5 on BC exam and used AP: 3.32

Morgan & Ramist, 1998
1996–99

**Comparison of AB exam vs Calc I**

University of Texas at Austin

Average grade in Calculus II:

Took Calc I at that university: 2.55

3–5 on AB exam and used AP: 2.98

Non-AP students chosen so that SAT scores are comparable for both sets of students

Dodd *et al*, 2002
## Fall of 1994

### Comparison of AB exam vs Calc I

<table>
<thead>
<tr>
<th>Average grade in Calculus II:</th>
<th>SAT-adjusted</th>
</tr>
</thead>
<tbody>
<tr>
<td>Took Calc I at that university:</td>
<td>2.43</td>
</tr>
<tr>
<td>3 on AB exam and used AP:</td>
<td>2.69</td>
</tr>
<tr>
<td>4 on AB exam and used AP:</td>
<td>2.90</td>
</tr>
<tr>
<td>5 on AB exam and used AP:</td>
<td>3.34</td>
</tr>
</tbody>
</table>


Morgan & Klaric, 2006
Fall of 1994

Comparison of BC exam vs Calc II


Average grade in Calculus III: SAT-adjusted

Took Calc II at that university: 2.50
3 on BC exam and used AP: 3.00 2.92
4 on BC exam and used AP: 3.45 3.35
5 on BC exam and used AP: 3.46 3.27

Morgan & Klaric, 2006
Conclusion:

Students who take AP Calculus in high school and do well on the exam are at least as well prepared for the next class as those coming out of the comparable college courses.

This is a very large group (about 145,000) of very talented students. We are not doing all we can and should to entice these students to continue mathematics and to prepare them for mathematically intensive majors.
Mainstream Calculus II Enrollments
(fall only for 2- & 4- year colleges and

students (thousands)

academic year

CBMS & College Board data
Calculus Before Grade 12

College Board data
BC exam,
8,818 in 2002
15,533 in 2007
76% increase

College Board data
American Math Competition survey, spring 2007, 3516 high schools

1582 reported at least some students take highest level of calculus offered before senior year.

1114 reported that at least some of these students take no mathematics in their senior year. In 203 of these schools, more than 50% of these students take no math in their senior year.

Reasons:

• Taking AP courses in other subjects - 580
• Intended major doesn’t require any more math - 458
• No courses available - 245
Conclusion:

Students completing calculus before their senior year are the best of the best, and their numbers are significant.

Colleges and universities have a responsibility to work collaboratively with the high schools to find ways to encourage these students to continue their study of mathematics and consider mathematically intensive majors.
Those who take calculus in high school but do not receive college credit for it consist of:

~100,000 take generic “calculus”;

~105,000 take AP calculus but not the exam;

105,000 take AP exam and score less than 3;

~30,000 score 3 or better on AP exam but do not receive credit or elect not to use it

Total: ~ 340,000
About 340,000 students arrived at college this past fall having taken calculus in high school but without bringing college credit for it.
About 340,000 students arrived at college this past fall having taken calculus in high school but without bringing college credit for it.

That is greater than the total number (325,000) of students who took Calculus I in all 2- and 4-year colleges and universities in the fall.
Conclusion:

Colleges and universities need to pay serious attention to these students. They constitute a very large proportion of those who seek mathematically-intensive majors.

Although we need more data on what happens to these students, simply treating them as though they have not seen calculus before does not work.
US Dept. of Ed. transcript study, high school class of 1992, 30% of those who took a course called “calculus” in high school took precalculus when they got to college. If extrapolated to current numbers, that would be over 150,000.

**Conclusion:**

We need to rethink what we do with students who take “calculus” in high school but are not “ready” for calculus when they get to college.
College enrollment in Calculus I has been constant at about half a million students per year over the past quarter century.

During this time, the number of students arriving with credit for Calculus I has gone from a negligible number to about $145,000 + 30,000 = 175,000$.

The top third of the Calculus I students have disappeared and been replaced by an equal number of students who would not have taken Calculus in 1982.

Probably about half of all students taking Calculus I in college took a calculus class in high school.
Conclusion:

If students in college Calculus I seem less well prepared today than they did 20 years ago, it is because we are looking at a different segment of the student population.

This is not a reason to lower expectations, but it is reason to reconsider what it is we really want these students to learn and how we can adjust our pedagogy to help them accomplish it.
Our solution at Macalester College:

An *Applied Calculus* course with emphasis on

• Reading and writing differential equations (including systems of dif eqs) and understanding the nature of their solutions

• Functions of several variables (including directional derivatives and a geometric treatment of Lagrange multipliers)

• An geometric introduction to linear algebra
Our solution at Macalester College:

A *Single Variable Calculus* course with emphasis on

• Techniques of calculus.

• The ability to work with differentiation and integration on a level that is both rigorous and conceptual.

• Taylor polynomials and approximations.
At 5-year intervals starting in 1990, CBMS has been tracking number of sections of mainstream Calculus I that use various markers of reform calculus:

• Use of graphing calculators
• Use of computer assignments
• Use of writing assignments
• Use of group projects

Results from 2005 were published last year.
Graphing Calculators

per cent of sections

year


PhD
MA
BA
2-year

CBMS data
Computer Assignments

<table>
<thead>
<tr>
<th>Year</th>
<th>PhD</th>
<th>MA</th>
<th>BA</th>
<th>2-year</th>
</tr>
</thead>
<tbody>
<tr>
<td>1990</td>
<td></td>
<td>5%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1995</td>
<td></td>
<td>10%</td>
<td>15%</td>
<td></td>
</tr>
<tr>
<td>2000</td>
<td>20%</td>
<td>40%</td>
<td>35%</td>
<td></td>
</tr>
<tr>
<td>2005</td>
<td></td>
<td>10%</td>
<td></td>
<td></td>
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CBMS data
Writing Assignments

per cent of sections

year


PhD MA BA 2-year

CBMS data
Use of online resources in mainstream Calculus I (2005):

PhD: 9%
MA: 2%
BA: 2%
2-year: 5%
Conclusion:

Reform fatigue is a serious issue. Those who have found reforms that work need to generate broad buy-in within their departments. This includes finding mechanisms (including the use of on-line resources) that assist faculty to fully engage students.
Summary Conclusions:

1. We need more current and comprehensive data about what happens to students as they cross the divide from high school to college.

2. We need to rethink calculus instruction and how to meet the needs of the constituencies that did not exist in anything like these numbers 25 years ago:
   - Those who arrive with credit for calculus;
   - Those who are retaking the calculus course they took in high school;
   - Those who studied calculus in high school but are not deemed ready for it when they get to college;
   - Those who would not have taken calculus 25 years ago.
More information and detailed references in my monthly *Launchings* columns at www.maa.org

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