Projects in Analysis
pdf files and links at
www.macalester.edu/~bressoud/talks
Rearranging the Harmonic Series:

\[1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots = \ln 2.\]

\[1 + \frac{1}{3} - \frac{1}{2} + \frac{1}{5} + \frac{1}{7} - \frac{1}{4} + \cdots = \frac{1}{2} \ln 2\]

\[1 + \frac{1}{3} + \cdots + \frac{1}{2r-1} - \left(\frac{1}{2} + \frac{1}{4} + \cdots + \frac{1}{2s}\right)\]
\[+ \left(\frac{1}{2r+1} + \frac{1}{2r+3} + \cdots + \frac{1}{4r-1}\right)\]
\[= \quad ?\]

Inverse Symbolic Calculator at
www.cecm.sfu.ca/projects/ISC/ISCmain.html

proofs can be done using
\[1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} = \ln n + \gamma + E(n)\]

where \(\lim_{n \to \infty} E(n) = 0;\)

or by using the power series expansion of \(\ln(1 + x).\)
What does \( \sqrt{2^{\sqrt{2^{\sqrt{2^{\cdots}}}}} \cdots} \) mean? How large is it? For what values of \( x \) does \( x \cdot x \cdot x \cdots \) converge?

see Knoebel, “Exponentials reiterated”

A sequence \( \{x_n\} \) of real numbers is said to be **Cesàro convergent** to \( x_0 \) if the sequence of its averages has the limit \( x \)

\[
\lim_{n \to \infty} \frac{x_1 + x_2 + \cdots + x_n}{n} = x_0.
\]

Prove that if \( \lim_{n \to \infty} x_n = x_0 \), then the sequence \( \{x_n\} \) is Cesàro convergent to \( x_0 \).

We can use the symbol

\[ x_n \xrightarrow{C} x_0 \]

to mean that \( \{x_n\} \) is Cesàro convergent to \( x_0 \). We say that a function \( f \) is **Cesàro continuous** at \( x = x_0 \) if

\[ x_n \xrightarrow{C} x_0 \quad \text{implies} \quad f(x_n) \xrightarrow{C} f(x_0). \]

Note that we have weakened the conclusion, but we have also weakened the hypothesis. Is every continuous function also Cesàro continuous? Is every Cesàro continuous function also continuous?