Can Fundamental Risks Explain the Foreign Exchange Risk Premium?

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Abstract
This paper tests whether the foreign exchange risk premium is primarily driven by fundamental macroeconomic risks. To avoid limitations with existing similar tests, both the premium and risk factors are independently measured and the decision is based on the significance of their correlation. All commonly discussed fundamental risks proposed by various studies are included in our test. Except that monetary risk and inflation risk are found to be related to currency premium in a few cases, none of the fundamental risks are consistently and significantly correlated with the currency premium. Furthermore, we implement a joint test that reveals a weak correlation between the currency risk premium and the fundamental macroeconomic risks. Therefore we conclude that fundamental risks do not appear to be the significant risk sources for the currency risk premium.

Keywords: fundamental risks, foreign exchange risk premium

JEL classification: F31, E44, G12, G15
1. Introduction

One of the most intriguing empirical results in the area of international finance is the phenomenon of forward premium puzzle. Under the familiar conditions of uncovered interest parity and rational expectations, the forward premium (i.e. the difference between the forward exchange rate and the current spot rate) should be an unbiased predictor for the future exchange rate change. Existing evidence shows, however, that the actual change in a spot exchange rate is poorly predicted by the forward premium. In fact, the implied prediction is seriously biased and often has the wrong sign.

Since the puzzle was initially identified by Fama (1984), many studies have provided explanations for the puzzle, and a large portion of them attributes the puzzle to time varying risk premium. Identifying the risk source of the premium and explaining its dynamics become an important foundation for the risk premium-based explanations of the puzzle. Various risk sources have been examined in growing literatures many of which claim their success on relating the fundamental risks with currency risk premium. Lustig and Verdelhan (2007) as well as Santisyand and Fornariz (2008), for instance, find that consumption risk is priced in the foreign exchange risk premium. Iwata and Wu (2006) document that output, inflation and monetary policies can explain the currency premium dynamics. Instead of real factors, Poghosyan and Kocenda (2007) report that monetary and fiscal policies are more significantly related to the risk premium in some transition economies in European Union. Earlier supporting empirical literature also includes Hollifiled and Yaron (2003), Garcia and Kichian (2000), Hu (1997), and etc.

Despite documented supportive empirical evidence, the implications of these results could be jeopardized by several limitations associated with the current literature.
First, the typical test in these papers is to regress risk premium on the conditional variances of risk factors, while the variances are usually estimated through a VAR-GARCH type of specification. Some papers (e.g. Hollifield and Yaron (2003), Iwata and Wu (2006), Poghosyan and Kocenda (2007)), however, include exchange rates in the estimation, which gives the estimated conditional variances of the innovations from the foreign exchange market that might not be directly associated with the fundamental risk factors. Thus, high explanation power provided by the estimated variances does not necessarily suggest that the currency premium is driven by the claimed risk factors.

Second, as argued by Burnside (2007), any proposed risk source should be significantly correlated with the risk premium. In this sense, lower significance level of risk betas reported by Burnside (2007) and Santis and Fornariz (2008) undermines the validity of R-square-based supportive evidence reported in Lustig and Verdelhan (2007). Third, some other papers (e.g. Bams (2004)) extract latent factors from fundamental risks and link them to currency premium. However, this type of research does not offer an economic interpretation to these factors. The risk source of currency premium remains in mystery even latent factors is found to be significantly related to the premium.

Objectively, the criteria to determine whether one risk factor is the source of the currency premium should at least include the following two aspects: first, risk premium and risk factors should be independently measured; second, significant covariance between the risk factors and the risk premium should be detected. As pointed out above, most existing papers are more or less vulnerable in these aspects. Burnside et al. (2006), however, conduct a test that satisfies both the requirements. They regress currency speculation payoff (i.e. risk premium) on fundamental risk factors and find no supporting
evidence. One limitation with their test is that they use level of fundamental factors rather than variances, which seems to be inconsistent with theoretical framework. Meanwhile they only focus on the monetary risks.

This paper has two purposes. First, due to the limitations with the current research, it is necessary to apply a methodology free of issues noted above to test whether fundamental factors drive the currency premium. Without positive evidence, any fundamental-risk-based explanation for the puzzle lacks a solid foundation. Second, as noted before, multiple risk sources have been mentioned in literature and include consumption risk (e.g. Lustig and Verdelhan (2007), Santis and Fornariz (2008)), output risk (e.g. Farhi and Gabaix 2008, Hollifiled and Yaron (2003)), monetary risks such as interest rate risk (e.g. Carlson and Osler (1999), Backus, Telmer and Foresi (2001)), money supply risk (e.g. Hollifiled and Yaron 2003) and inflation risk (e.g. Iwata and Wu (2006)) and finally financial risks such as stock market return (e.g. Lustig and Verdelhan (2007)). The significance and explanation power of all these risks will be examined in this paper. These two features also differentiate our paper from other similar empirical studies.

In theory, risk premium in the financial market is the compensation for bearing the risk. In the foreign exchange market, however, it might not be true that investors should be rewarded for holding foreign assets simply because to hold them requires that they bear some foreign exchange risks.\textsuperscript{1} As Engel (1996) emphasizes, most foreign exchange risk is diversifiable. Investors should not be rewarded for taking on

\textsuperscript{1} Indeed, there is almost a logical inconsistency in this logic. If Americans were to receive a risk premium for holding foreign assets, the expected return on foreign assets would exceed the expected return on dollar assets. If foreigners were to be rewarded for holding dollar assets, the reverse would have to be true.
unnecessary risk. In modern models of returns on financial assets, a risk premium is awarded only when the return on an asset covaries with some benchmark (such as the return on the market portfolio) that makes risk undiversifiable. Therefore, the foreign exchange risk premium depends on the relative riskiness of domestic and foreign nominal assets.

To test this relationship, we first estimate the domestic and foreign fundamental risks without involving exchange rate data, take a difference of two countries’ risks to obtain relative riskiness, and then regress currency risk premium on the relative riskiness to test the significance of risk factors. In both individual risk tests and a joint test, we find no significant risk betas, which leads to our conclusion that fundamental risks do not appear to be the significant risk sources of the currency risk premium.

The remaining of the paper is arranged as below. Section 2 presents a general theoretical framework that motivates our test. Section 3 tests individual risk factors based on a ARMA-GARCH specification and Section 4 applies a joint test of all risk factors based on a Markov regime-switching model. Section 5 concludes.

2. The Theoretical Framework

Intuitively, the foreign exchange risk premium is determined by the relative riskiness of the two countries’ economies. Such a relationship can be derived explicitly in a conventional asset pricing framework. Assume that $M_{t+1}$ is the domestic pricing kernel and $R_{t+1}$ is the gross return of domestic financial assets at time $t+1$, we have:

$$E_t[M_{t+1}, R_{t+1}] = 1$$  \hspace{1cm} (2.1)
Where $E_t$ is the conditional expectation based on information up to time $t$. For the assets denominated in foreign currency that can be purchased by domestic investors, we have:

$$E_t[M_{t+1} \frac{W_t}{W_{t+1}} R^*_t] = 1$$ (2.2)

Where $W_t$ is spot rate denoted as foreign currency per US Dollar and $R^*_t$ is the gross return of foreign financial assets. Meanwhile, for foreign investors, there is a corresponding foreign pricing kernel $M^*$ that satisfies:

$$E_t[M^*_t R^*_t] = 1$$ (2.3)

Both Bansal (1997) and Backus et al (2001) (Proposition 1) have proved the following relationship:

$$\frac{W_{t+1}}{W_t} = \frac{M_{t+1}}{M^*_t}$$ (2.4)

The logarithm format of equation (2.4) is:

$$w_{t+1} - w_t = m_{t+1} - m^*_t$$ (2.5)

Where, $w_t = \log(W_t)$ and $m_t = \log(M_t)$, $R^f_t$, the risk free return at time $t$ with 1 period maturity should satisfy equation (2.1): $R^f_t E_t[M_{t+1}] = 1$, which implies $R^f_t = 1/ E_t[M_{t+1}]$.

Take a log on both sides, we have:

$$\log(R^f_t) = -\log(E_t, M_{t+1})$$ (2.6)

Gross return rates $R$ can be written as $R=1+i$, where $i$ is net return rate (interest rate). Using the approximation that $\log(1+x) = x$ if $x$ is a small number (usually less than 0.1), we have:

$$\log R^f_{t,1} = \log(1 + i_{t,1}) = i_{t,1}$$ (2.7)
Combining (2.6) and (2.7) gives:

\[ i_{t,1} = -\log(E_t M_{t+1}) \]  

(2.8)

Backus, Foresi and Telmer (2001) show that:

\[ i_{t,1} = -\log(E_t M_{t+1}) = -[E_t (\log M_{t+1}) + \sum_{j=2}^{\infty} \kappa_j / j!] = -[E_t m_{t+1} + \sum_{j=2}^{\infty} \kappa_j / j!] \]  

(2.9)

where \( \kappa_j \) is the \( j \)th cumulant for the conditional distribution of \( m \). The cumulants are closely related to the conditional moments of the distribution, i.e. \( \kappa_2 = \text{Var}_t (m_{t+1}) \) on the second order where \( \text{Var} \) denotes variance.

Taking conditional expectation of equation (2.5), we obtain:

\[ E_t (w_{t+1} - w_t) = E_t m_{t+1} - E_t m_{t+1}^* \]  

(2.10)

From both domestic and foreign versions of the equation (2.9), we may have:

\[ E_t (w_{t+1} - w_t) = E_t m_{t+1} - E_t m_{t+1}^* = i_{t,1}^* - i_{t,1} + \sum_{j=2}^{\infty} \frac{(\kappa_j^* - \kappa_j)}{j!} \]  

(2.11)

Given the Covered Interest Parity condition:

\[ i_{t,1}^* - i_{t,1} = f_{t,1} - w_t \]  

(2.12)

Where \( f_{t,1} \) is the logarithm of forward rate at time \( t \) with maturity of 1 period. Plugging equation (2.12) into equation (2.11) gives:

\[ E_t (w_{t+1} - w_t) = i_{t,1}^* - i_{t,1} + \sum_{j=2}^{\infty} \frac{(\kappa_j^* - \kappa_j)}{j!} \]  

(2.13)

And the risk premium, therefore, is:

\[ \sum_{j=2}^{\infty} \frac{(\kappa_j^* - \kappa_j)}{j!} \]  

(2.14)

If we assume log-normally for all variables, we are left with only the second-order term:
This states that risk premium is determined by the relative riskiness between foreign and domestic economies.

3. The Test of Individual Risk Factors

Intuitively, pricing kernel should be related to idiosyncratic risk factors, so the variance of pricing kernel can be approximated by the variance of risk factors. In individual risk test, we use a ARMA($r,m$)-GARCH($p,q$) specification as below to estimate such variances first:

\[
\begin{align*}
\sigma_i^2 &= \sigma + \sum_{j=1}^q \alpha_i \epsilon_{i-j}^2 + \sum_{j=1}^p \beta_j \sigma_{i-j}^2, \\
\epsilon_i \mid I_t &\sim N(0, \sigma_i^2)
\end{align*}
\]

Where $y_i$ is an individual risk factor, $I_t$ is the information set at time $t$. Based on the Bayesian information Criterion ($BIC$), optimal lag specification is determined as $r = 1, m = 2, p = 1, q = 1$.

Then, equation (2.15) can be tested by regressing risk premium on the difference between the conditional variances of foreign and domestic risk factors as below:

\[
\begin{align*}
rp_t = \lambda_0 + \sum_{i=1}^N \lambda_i (\hat{\sigma}_{t,i}^2 - \hat{\sigma}_{t,i}^2) + \eta_t
\end{align*}
\]

Where, $rp_t$ is risk premium for each currency pair, $\hat{\sigma}_{t,i}^2, \hat{\sigma}_{t,i}^2$ are the estimated variance of foreign and domestic risk factors $i$ at time $t$ respectively. $N$ is the total number of risk factors included in the test. $\eta_t$ is standard error item. A series of $\lambda$ are constant.
coefficients. The source of the risk premium can be identified by the significance of the coefficient of risk factors.

Our foreign exchange data consist of monthly spot and forward rates of the US dollar in Canadian Dollar, Japanese Yen and British Pounds. The maturity of the forward rates is 1-month. For January 1973 through February 1985, the data were from Harris Bank. The data from March 1985 to July 1997 are obtained from DataStream, originally generated by the National Westminster Bank. The earlier versions of these data were also examined by Roll and Yan (2000). The data from August 1997 to November 2004 are extracted from new edition of DataStream. Suppose $w_{t+1}$ is logarithm of spot rate and $f_{t,1}$ is logarithm of forward rate, monthly risk premium is defined as:

$$rp_t = w_{t+1} - f_{t,1}$$

Fundamental risk variables covered in this paper include consumption risk, output (technology) risk, monetary risk and equity market risk. Correspondingly, for each country we have collected the data of consumption, GDP, money supply, short term interest rate, inflation rate and stock market index to estimate these risks. Consumption risk factor is represented by the growth rate of consumption. Output risk factor is approximated by the growth rate of GDP. Money supply growth rate, short term interest rate and inflation rate are all used to reflect monetary risk. Equity market risk is the return rate of stock index. Thus the total number of fundamental risk factors examined in this test is 6, i.e. $N=6$ in equation (3.2). The quarterly data covering the first quarter of 1973 through the last quarter of 2004 are obtained from OECD macro database².

² The highest frequency of the data in OECD database is quarterly.
A practical issue to examine the data describe above is that the data of foreign exchange risk premium and fundamental risk factors do not have the same frequency. The former is monthly while the latter is quarterly. We converted the monthly risk premium to quarterly observation by taking average of three month’s risk premium in a quarter.

Results of the regression (3.2) are reported in Table 1. As one of few significant results, Money supply is found to be significantly correlated with JPY/USD risk premium. This result seems to be consistent with some other papers (e.g. Iwata and Wu (2006)) that report monetary policy to be the source of the premium. And also, inflation risk is also found to be correlated with currency premium for CAD/USD at a 10% significance level. Overall, however, none of the fundamental risks examined in the test generate consistent significant coefficients across currency pairs. This result suggests that the explanation of the fundamental risks for the currency premium is low.

This test is straightforward and intuitive; however, its limitations cannot be ignored. First the variance of risk factors is just an approximation for the variance of pricing kernel. Second, the variation of the second moment of pricing kernel can be attributed not only to the change of the risk factors themselves but also the change of loadings, while this test assumes constant loadings. The next session will conduct a more sophisticated test that is free of these problems.

4. The Joint Test

Based on discussions in Section 2, the foreign exchange risk premium ought to be related with the difference between two conditional variances of pricing kernels in both
domestic and foreign countries as shown in equation (2.15). Apparently, if this equation holds, the difference of fundamental risks \( (\text{VAR}_t (m^*_{t+1}) - \text{VAR}_t (m_{t+1})) \) will vary considerably and co-vary with the interest rate differential in the ‘right’ direction so that the forward premium puzzle will be resolved. However, the empirical test of (2.15) requires estimating the fundamental risks which are not directly observable. Section 3 approximates the fundamental risks by studying each important macroeconomic variable respectively and sees if there is any close relationship between the risk premium and each individual risk factor. Given potential interactions between risk factors it would be more desirable to implement a joint test to take into account these interactions.

Some previous studies have insightfully explored the information contained in bond markets so as to indirectly extract the fundamental risks to test (2.15) and discuss the implications to forward premium puzzle, e.g. see Backus et al (2001) and Wu (2007). We take a different approach instead by directly specifying a particular pricing kernel in terms of macroeconomic fundamentals based on discussions of the relevant general equilibrium models in foreign exchange literature (see e.g. Engel (1996) and Burnside et al (2006)) and assume the pricing kernel (in log term) is a linear function of the fundamental macroeconomic risk factors:

\[
m_{rel} = \beta_0 + \beta_1 g_{c,rel} + \beta_2 g_{y,rel} + \beta_3 \pi_{rel} + \beta_4 r_{a,rel}
\]

(4.1)

Where, \( g_c \) is the real consumption growth, \( g_y \) the real output growth, \( \pi \) the inflation rate, and \( r_a \) is the wealth portfolio return, approximated by the equity market return. In principle the conditional variance of pricing kernel \( \text{Var}_t (m_{rel}) \) should be constructed by estimating the risk loadings \( \beta \)'s and the risk matrix of \( \text{Var}_t (g_{c,rel}, g_{y,rel}, \pi_{rel}, r_{a,rel})' \)
respectively. Alternatively a more convenient approach, we shall employ below, is to take advantage of the implicit pricing relationship between the risk free rate and the fundamental variables to estimate $Var_r(m_{t+1})$ as a whole.

The one-period risk free rate denoted by $r_f$ satisfies:

$$E_r(\exp(m_{t+1} + r_{f,t+1})) = 1$$  \hfill (4.2)

Assuming log-normality for the pricing kernel, we may have:

$$r_{f,t+1} = -E_r(m_{t+1}) - Var_r(m_{t+1})/2$$  \hfill (4.3)

Using the specification (4.1) we derive the following regression:

$$r_{f,t+1} = -0.5Var_r(\eta_{t+1}) - \beta_0 - \beta_1g_{c,t+1} - \beta_2g_{y,t+1} - \beta_3\pi_{t+1} - \beta_4r_{a,t+1} + \eta_{t+1}$$  \hfill (4.4)

If we let $\tilde{g}_{t+1} = (g_{c,t+1}, g_{y,t+1}, \pi_{t+1}, r_{a,t+1})'$ and $\tilde{\beta} = (\beta_0, \beta_1, \beta_2, \beta_3, \beta_4)'$, (4.4) becomes:

$$r_{f,t+1} = -0.5Var_r(\eta_{t+1}) - (1, \tilde{g}_{t+1}')\tilde{\beta} + \eta_{t+1}$$  \hfill (4.5)

Where, $\eta_{t+1} = m_{t+1} - E_r(m_{t+1})$ is normally distributed and its time-varying variance $Var_r(\eta_{t+1})$ summarizes the fundamental risks which are related with the risk premium.

Such a regression as (4.5) is a typical one resulting from the rational expectation model, e.g., see Gali and Gertler (1999) in estimating the New-Keynesian Phillips curve or Clarida, Gali and Gertler (2000) in estimating the forward-looking Taylor rule. Since in these regressions regressors and error variable are correlated, one needs to instrument the regressors first and then estimate the main equation (4.5) in two steps. Notice that due to the convexity adjustment the variance of error variable also enters the mean equation. Since we are dealing with the quarterly macroeconomic data, the time-varying variance of error variable may be appropriately modeled as a Markov regime-switching process originally proposed by Hamilton (1989) and discussed extensively in Kim and Nelson.
At the same time the risk loading parameters $\beta's$ are also likely to experience frequent regime shifts. Kim (2009) develops a two-step estimation method to deal with such an Instrumental Variable estimation which allows for regime switching of parameters. Applying Kim’s estimation method, we can obtain an estimate of the shifting variances of pricing kernels for both domestic countries and foreign countries and ultimately use them to test (2.15).

Assume that a latent variable $S_t$ governs the state of the economy and $S_t$ is a $J$-state first-order Markov switching process. We set up the estimation framework as below:

$$r_{j,t+1} = -0.5\sigma_{\eta,S_{t+1}}^2 - (1, \tilde{g}_{t+1}^{\prime})\tilde{\beta}_{s_{t+1}} + \eta_{t+1} : \eta_{t+1} \sim N(0, \sigma_{\eta,S_{t+1}}^2)$$ (4.6)

$$\tilde{\beta}_{s_{t+1}} = \tilde{\beta}_1 S_{t+1} + \tilde{\beta}_2 S_{2t+1} + \ldots \tilde{\beta}_J S_{Jt+1}$$ (4.7)

$$\sigma_{\eta,S_{t+1}}^2 = \sigma_{\eta_1}^2 S_{t+1}^2 + \sigma_{\eta_2}^2 S_{2t+1}^2 + \ldots \sigma_{\eta_J}^2 S_{Jt+1}^2$$ (4.8)

$$S_{jt+1} = \begin{cases} 1, \text{if } S_{t+1} = j; j = 1,2,\ldots,J \\ 0, \text{otherwise}, \end{cases}$$ (4.9)

Where, the transition probabilities are:

$$Pr(S_{t+1} = j \mid S_t = i) = p_{ji}, \text{ and } \sum_{j=1}^J p_{ji} = 1 \text{ for any } i.$$ (4.10)

And $P$ is the $J \times J$ matrix of transition probabilities.

We follow previous studies such as Gali and Gertler (1999) to use lags of $\tilde{g}_{t+1}$ as instruments. Therefore the first-stage equation is literally a VAR($q$) system:

$$\tilde{g}_{r+1} = Z'_{t+1} \tilde{\gamma} + \Sigma_{t+1}^{1/2} \tilde{\epsilon}_{t+1}$$ (4.11)

$$\left[ \tilde{\epsilon}_{t+1} \right] \sim i.i.d.N\left(0, \begin{pmatrix} I_q & \rho \sigma_{\eta,S_{t+1}} \\ \rho' \sigma_{\eta,S_{t+1}} & \sigma_{\eta,S_{t+1}}^2 \end{pmatrix} \right)$$ (4.12)
Where, $Z_{t+1} = I_d \otimes z_{t+1}$ and $z_{t+1} = (1, g', \ldots, g'_{t+q})'$, a $(1 + 4q) \times 1$ vector with $q$ being the VAR order, which is chosen to be 4 since we are using the quarterly data here. The correlation vector $\rho \neq 0$ raises the issue of endogeneity and it may be tested directly in the estimation process as shown in Kim (2009).

In principle, the parameters in the first-stage equation (4.11) could also be subject to regime shifts. However, since the VAR system is 4-dimensional which involves many parameters to estimate, e.g., for the case of $q = 4$ there will be about 78 parameters to estimate for each state, for the sake of estimation stability we choose to have invariant parameters in the VAR.

As in Kim (2009) we take a Cholesky decomposition on the variance matrix in (4.12):

$$\begin{bmatrix}
\bar{e}_{t+1} \\
\eta_{t+1}
\end{bmatrix} =
\begin{bmatrix}
I_d & 0 \\
\rho' \rho \sigma_{\eta,S_{t+1}} & \sqrt{1 - \rho' \rho \sigma_{\eta,S_{t+1}}}
\end{bmatrix}
\begin{bmatrix}
v_{1t+1} \\
v_{2t+1}
\end{bmatrix}
\sim N\left(0, \begin{bmatrix} I_d & 0 \\ 0 & 0 \end{bmatrix}\right) \tag{4.12}
$$

As a result, the main equation (4.6) becomes:

$$r_{f,t+1} = -0.5 \sigma^2_{\eta,S_{t+1}} - (1, g'_{t+1}) \bar{\beta}_{S_{t+1}} + \bar{e}_{t+1}' \rho \sigma_{\eta,S_{t+1}} + v_{2t+1}, v_{2t+1} \sim i.i.d. N(0, (1 - \rho' \rho) \sigma^2_{\eta,S_{t+1}}) \tag{4.13}
$$

Where, $\bar{e}_{t+1} = \Sigma_e^{-1/2} (\bar{g}_{t+1} - Z'_{t+1} \bar{y})$. Since the error variable $v_{2t+1}$ is orthogonal to regressors in the estimation equation (4.13), Hamilton (1989) filter is readily applicable to construct the likelihood function provided estimated values of $\bar{e}_{t+1}$, which can be obtained in the first-stage regression in (4.11). After the parameter estimates are obtained we can apply Kim’s (1994) smoothing algorithm to calculate the smoothed estimate of the fundamental risks $E(\sigma^2_{\eta,t+1} \mid I_T)$ for each time point. Next, we briefly describe the estimation steps and interested readers are referred to Kim (2009) for more details.
In the first step, estimate the equations in the VAR system as given by (4.11).

Collect the standardized estimated residuals: 
\[ \hat{\epsilon}_t = \hat{\Sigma}_t^{-1/2} (\tilde{g}_t - Z'_{\hat{\psi}}) .\]

In the second step, plug \( \hat{\epsilon}_t \) into equation (4.13) and maximize the following log likelihood function by integrating out the latent variable \( S_t \):
\[
\ln[f(R_{f,t} | G_t; \theta_1, \hat{\theta}_2)] = \sum_{i=1}^{T} \ln \left[ \sum_{S_{i+1}} f(r_{f,i} | S_i, R_{f,i-1}, G_t; \theta_1, \hat{\theta}_2) f(S_i | R_{f,i-1}, G_t; \theta_1, \hat{\theta}_2) \right]
\]
(4.14)

Where, \( R_{f,i} = [r_{f,i} \quad r_{f,i-1} \quad \ldots \quad r_{f,1}]' \), \( G_t = [g_1 \quad g_2 \quad \ldots \quad g_i]' \), \( \theta_1 = [\hat{\beta}', \ldots, \hat{\beta}', \hat{\rho}', \sigma_{\eta_j}', \ldots, \sigma_{\eta_j}']' \) \( vec(P)' \). And the filtered probabilities are given by:
\[
f(S_{t+1} = j | R_{f,t}, G_t; \theta_1, \hat{\theta}_2) = \sum_{i=1}^{T} \Pr[S_i = i | R_{f,t}, G_t; \theta_1, \hat{\theta}_2] \Pr[S_{t+1} = j | S_i = i]
\]
(4.15)

The updating of the probabilities is:
\[
\Pr[S_i = j | R_{f,t}, G_t; \theta_1, \hat{\theta}_2] = \frac{f(r_{f,i} | S_i = j, R_{f,i-1}, G_t; \theta_1, \hat{\theta}_2) f(S_i = j | R_{f,i-1}, G_t; \theta_1, \hat{\theta}_2)}{f(r_{f,i} | R_{f,i-1}, G_t; \theta_1, \hat{\theta}_2)}
\]
(4.16)

The conditional density is:
\[
f(r_{f,i} | S_i = j, R_{f,i-1}, G_t; \theta_1, \hat{\theta}_2) = \frac{1}{\sqrt{2\pi(1-\hat{\rho}^2)\sigma_{\eta_j}^2}} \exp\left\{ -\frac{1}{2(1-\hat{\rho}^2)\sigma_{\eta_j}^2} \left( r_{f,i} + 0.5\sigma_{\eta_j}^2 (1+\hat{g}_t')\hat{\beta}_j - \hat{\epsilon}_t, \hat{\rho}\sigma_{\eta_j} \right)^2 \right\}
\]
(4.17)

The maximization of the log likelihood function in (4.14) results in a set of parameter estimates based on which standard errors of estimates could be computed.

Table 2 summarizes the first step estimation, i.e., a 4-variable VAR system with 4 lags. Given identical regressors for each equation, one-by-one OLS estimation gives
consistent and efficient estimator under regular conditions. The goodness of fit equation by equation also gives a sense about which equation has relatively better forecasting ability. As demonstrated by the results in Table 2 there are a fair amount of predictability for all equations and for all four countries considered here.

Using the estimated residuals \( \hat{\epsilon}_t = \hat{\Sigma}_e^{-1/2} (\hat{\tilde{y}}_t - \tilde{Z}_t^\prime \hat{\eta}_t) \) from the first step estimation, estimate the parameters in the second-stage equation in (4.4). Both 2 regimes and 3-regimes are tried and 2 regimes seem to fit the data sufficiently well. The estimation results are summarized in Table 3.

Based on the estimates in Table 3, it seems that the estimated differences between high risk regime and low risk regime are more pronounced in US and CANADA than in JAPAN and UK. Next, we compute and plot the smoothed estimates of the risk factors \( \hat{\sigma}_{q,t}^2 \) for each country in Figure 1 through 4 based on the formula:

\[
\hat{\sigma}_{q,t}^2 = \Pr(S_t = 1 \mid I_t) \cdot \hat{\sigma}_{q,1}^2 + \Pr(S_t = 2 \mid I_t) \cdot \hat{\sigma}_{q,2}^2.
\]

Here the smoothed probability estimates are obtained by following Kim’s (1994) algorithm. It appears that all countries have experienced a major risk reduction in the early 90s.

To test (2.15), we regress \( (w_{t+1} - f_{t,1}) \), which is equal to ex-ante risk premium plus noise: \( E_r (w_{t+1} - f_{t,1}) + (w_{t+1} - E_r (w_{t+1})) \), directly on the estimated relative riskiness \( \hat{r}_t = \hat{\sigma}_{q,t}^2 - \hat{\sigma}_{q,t}^2 \) without assuming any systematic dynamics for the unobserved risk premium except that it is uncorrelated with the noise:

\[
r_t = a + b \cdot \hat{r}_t + \nu_t
\]

We rely on the previous dataset of the 1-month foreign exchange forward rate at time \( t \) \( f_{t,1} \) and the spot rate \( w_t \), all in log terms, for Canada/US, Japan/US and UK/US. To
facilitate the comparison with fundamental risks we average monthly estimates to obtain quarterly data. The estimations results are laid out in Table 5.

As suggested by the results in Table 5, the explanatory power of the fundamental risks to the foreign exchange risk premium is quite low. Our findings suggest that the fundamental macroeconomic risks do not appear to be sufficient in explaining the foreign exchange risk premium. However, one surely needs to interpret our results with caution due to several restrictions we place to implement our tests. In the meantime, if our assumptions are not violated in such a dramatic way which will invalidate our efforts, the implication of this weak connection between the fundamental risk and the foreign exchange risk premium is that it would be very hard to resolve the forward premium puzzle solely based on the macroeconomic fundamental risks.

5. Conclusion

Identifying risk sources of currency premium is essential for risk premium-based explanations for the forward premium puzzle. Multiple fundamental risks have been reported by growing empirical studies to be able to explain the dynamics of currency premium. Realizing some limitations associated with the current literature, we conduct a test to examine the explanation power of various fundamental factors in this paper.

Foreign exchange risk premium is believed to be determined by the relative riskiness of two countries. This simple intuition becomes the basis of our test. We start with estimating risks of each country and then test correlation between risk premium and relative riskiness. Multiple fundamental risks, including consumption risk, output risk, monetary risk, inflation risk and financial risk, are examined in the test. Results, however,
are not promising. None of these frequently discussed fundamental risks are found to be consistently and significantly correlated with foreign exchange risk premium, which suggests that fundamental risks are not the risk source of the currency premium.

Our results put fundamentals-based risk premium explanations in jeopardy. To keep the premium explanations alive, our results suggest the risk source search turn to non-fundamental information or irrational response to fundamental information. Meanwhile, our results reinforce the chance of other angles such as microstructure approach (e.g. Burnside et. al. (2007)) to tackle this well-known puzzle.

Rejection of the test in our paper does not imply failure of the intuition that relative riskiness determines the currency premium. The results in our paper only imply that fundamental risks are not the significant sources of such riskiness. Then what are the sources becomes a natural question. We think of two possible directions for further research. First, given unsuccessful evidence from fundamental risks, non-fundamental risks deserve more attention. Second, as argued by McCallum (1994), risk premium might not be the premium for risks in the foreign exchange market at all. He states that it might "represent time-varying aggregation or other effects". If it is the case, identifying the “other effects” is probably the other direction to go.
REFERENCES


Table 1: Estimation Results of Individual Risk Test

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>CAD/USD</th>
<th>JPY/USD</th>
<th>GBP/USD</th>
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</thead>
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<td></td>
<td></td>
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<td>(0.46)</td>
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<td>-0.1900</td>
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<td>(-0.78)</td>
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*Note: Numbers in parentheses are t-statistics;*
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<th>Variables</th>
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*Note:* Each variable starts with its country followed by the macro variable acronym: cmp-nominal consumption growth, gdp-nominal GDP growth, inf-inflation rate, ret-stock return. All data are from the OECD database. The sample period is 1973Q1 – 2004Q4.
### Table 3: The Second Step Estimation

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**Note:** Numbers in parentheses are standard errors; LLF is the maximized value of the log likelihood function.
Table 4: The Estimation Result of the Foreign Exchange Risk Premium

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<th>Parameters</th>
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<th>JPY/US</th>
<th>UK/US</th>
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<td>( q )</td>
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<td>(0.0247)</td>
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<tr>
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<td>-0.0773</td>
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<td>(0.1974)</td>
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<td>(0.0544)</td>
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</table>

Note: Numbers in parentheses are standard errors. The dataset is monthly from 1973.4 to 2004.11.
Table 5: Regression of Risk Premium on Relative Riskiness

<table>
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<tr>
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<th>UK/US</th>
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</thead>
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<td>-0.0040</td>
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</tr>
</tbody>
</table>

*Note:* Numbers in parentheses are standard errors.
Figure 1: Smoothed Estimate of the Fundamental Risk – US
Figure 2: Smoothed Estimate of the Fundamental Risk – CANADA

CANRISK

Figure 3: Smoothed Estimate of the Fundamental Risk – JAPAN

JPYRISK

1.9  2.0  2.1  2.2  2.3  2.4  2.5  2.6  2.7  2.8

Figure 4: Smoothed Estimate of the Fundamental Risk – UK

UKRISK