The basic relationship between the goal point, the model point, and the residual is given by a right triangle, as shown in the figure.

The residual is always perpendicular to the model subspace. The response is the hypotenuse of the right triangle.

This relationship holds no matter how large \( N \) is because the fitted model is always a single vector (which lives in the subspace spanned by the model terms) and the response is always a single vector. So the response vector and fitted model vector define a plane. The residual, being a linear combination of these two vectors, is also in that plane.

The square length of the fitted model vector is just the sum of squares of the fitted values. The square length of the residual vector is just the sum of squares of the residuals. Since there is a right triangle relationship, the square length of the response variable, that is, the sum of squares of that variable, must always be equal to the sum of the square lengths of the fitted model vector and the residual vector.

Choose some data set that we have used in class. Construct some model and verify that the above statements are true.

1. Give the computer commands you used to show that the sum of squares of the residual plus the sum of squares of the fitted model equals the sum of squares of the response variable.

2. Give the computer commands you used to show that the residual vector is orthogonal to the fitted model vector. ("Orthogonal" means "perpendicular.")