Reviews

Fusaro, B.A., and P.C. Kenschaft (eds.). 2003. *Environmental Mathematics in the Classroom*. Washington, DC: Mathematical Association of America; viii + 255, $50 (P). ISBN 0–88385–714–6.

"Environmental mathematics seeks to marry the most pressing challenge of our time with the most powerful technology of our time—mathematics." The sentence that introduces this little book promises important things to come. The chapters that follow provide 13 stand-alone and independently written case studies of precalculus mathematics applied to . . . well, it's not quite clear what.

Do not look toward this book for a systematic coverage of environmental topics, or even for an introduction to the high points of environmental concern. The chapters cover a scattering of subjects, only some of which are clearly primarily environmental in nature:

- environmental remediation (oil tanker spills and oil-tank overflowing),
- the spread of pollutants (ground water movement, sulphur dioxide from power plants), and
- the assessment of the economic value of shared environmental resources.

Other chapters are about

- physiology (lead poisoning in humans),
- genetics (albinism in bison),
- ecology (three chapters, each introducing population growth models),
- and the physics of water condensation in rising air.

The topics in the book are not the pressing environmental challenges of global warming, biodiversity, deforestation, food and hunger, rapid industrial growth, and exhaustion of energy resources. Nor do the chapters give mathematics that might be brought to bear on these challenges. The book's introductory sentence has it backwards: The pressing challenge that this book addresses is not safeguarding the environment but how to teach mathematics. The powerful technology is not mathematics but a pedagogical technique: the use of applications that students care about.

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One chapter gives some useful suggestions to instructors on how to mine newspapers and magazines for environmental stories that can illustrate some mathematical point. Several other chapters introduce some physical or biological principles for which mathematics is the natural language—e.g., groundwater flow, gene frequencies. These can be interesting, but most readers would be uncomfortable with them unless they already have the physics or biology background.

The remaining chapters are mainly about "modeling." One helpful definition of a model, not given in the book, is "a representation for a particular purpose." Since the purpose of these models is to teach some mathematical approaches (such as finite-difference equations) and to demonstrate some mathematical phenomena (such as exponential growth), it shouldn't be surprising that the models have been constructed in a way that avoids environmental realism and neglects critical examination of the weaknesses in the models. The student studying these models must come away with the impression that models are completely separate from the real world and that modelers are entitled to make up anything they want: modeling as fiction writing. One section in the book is entitled, "Six population rules in Mathland" (p. 100).

There is a charming history in mathematics of writing in terms of imaginary landscapes. Population growth in Mathland is well within the tradition of Edwin A. Abbott's examination of the social structure of Flatland [1992]. Fairy tales help people to impose order on the world; mathematical tales help people to see the sometimes unexpected consequences of their assumptions and to break out of their intuition. Both kinds of tales teach people categories for analyzing the real world.

Exponential growth and decay, transport between compartments, sigmoidal growth, multistability, and oscillation are all important concepts for understanding populations; and fairy tales have been used to teach them. (My favorite examples are the emergence of oscillations in simple predator-prey model systems and a beautiful illustration of multistability in May [1977].) It's helpful that libraries don't shelve fairy tales under nonfiction; it would be similarly helpful if *Environmental Mathematics in the Classroom* made a clear distinction between teaching population-motivated dynamics and talking about the environment.

Several chapters present a mathematical or physical concept in an accessible and compelling way. Particularly noteworthy are chapters on groundwater hydrology and age-structured population models. The worst chapters use the "environmental" setting as a pretext for ad hoc fitting of polynomials. There is no genuine environmental content here and very little statistics either. In one chapter, the fitted model coefficients are inexcusably reported to 16 digits, with no mention of the idea of a margin of error.

The style varies markedly from author to author; the intended readership slips back and forth between students and their teachers:

- "Logarithms can do 'magic'" (p. 104).

- "When we have an over-determined equation of this sort, the least-squares solution can be calculated by multiplying both sides by the transpose of the matrix, then solving the resulting system" (p. 153).
- As an example of gratuitous mathematics, we have even: "Indexed by pairs of indices (k, l), the following subsections may be read in any nondecreasing order of the sum $k + l, \ldots$ " (p. 216).

A book that states, "[M]athematics is playing a key role in solving all kinds of environmental problems" (p. 39) should give at least one real-world example of this. This book doesn't. This deficiency is completely unnecessary: Most important environmental policy decisions are based on mathematical models and their projections, from global population to the effects of environmental toxins. Much of what is perceived as policy dysfunction stems from policy makers who don't know how to figure out whether to take the models seriously and from a politically active population who uncritically accept doom-and-gloom projections and conspiracy theories. Some well-crafted fairy tales would come in handy here to show how intuition can be wrong and complacency misplaced. Also helpful would be some nonfictional evaluation of the important models, to illustrate how a spread of predictions should not always lead to the dismissive evaluation, "It's all just a matter of opinion."

References

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Brannan, David A., Matthew F. Esplen, and Jeremy J. Gray. *Geometry*. New York: Cambridge University Press, 1999; xi + 497, $37. ISBN 0–52159787–0.

Farin, Gerald, and Dianne Hansford. *The Geometry Toolbox for Graphic and Modeling*. Natick, MA: A K Peters, 1998; xv + 288, $49. ISBN 1–56881–074–1.

Euclid's geometry of the plane, a model for axiomatic theories, makes up the most celebrated part of his *Elements*. It is also purely synthetic. Synthetic geometry has an elegance and a beauty that will always be especially satisfying to students and professors alike.

Analytic geometry, on the other hand, rarely provides such breathtaking proofs as does synthetic geometry, but it is more cohesive and lends itself to systematic methods. It thus lends itself to linear algebra, group theory, and Felix Klein's Erlangen program. It is most often the default geometry of applications (the applied geometric problem du jour is computer graphics). Historically, analytic geometry made its first appearance perhaps with Nicole Oresme in the fourteenth century but really got going with Descartes and Fermat in the seventeenth century.

The two books reviewed here are both on analytic geometry. They serve different readers with different needs; about the only thing that they have in common is that both books are very useful pedagogical tools and are valuable additions to the literature.

Brannan et al.'s *Geometry* assumes a knowledge of linear algebra and group theory, although there are brief appendices on them. Realistically, the group theory is dispensable. (If one merely points out that a given algebraic structure comprises a group and makes little use of group results, then group theory is not a serious requirement. This book, like some others on geometry, does not pursue group theory to any real degree.) There is negligible use of calculus. The book follows pedagogical conventions of the Open University, which govern such issues as the number of sections per chapter. I do not know if this makes any difference, but what does is that the book was written with great care. The quality of the exposition, the illustrations, and the organization are first-rate and serve to make the book perfect for self-study. Similarly, there are many exercises, a great chunk of which have worked answers in the back. The book is an excellent choice for the classroom.

The book is to some extent dedicated to Klein's Erlangen program and as such it is similar in spirit and style to Henle [1997]. I still like Henle a great deal, in fact as much as when I reviewed it [Cargal 1999]; but if I had to choose between the two books, I would choose Brannan et al. A shorter more group-theoretic text by Ryan [1986] is worth a look and has merit (but makes no mention of the Erlangen program). Mumford et al. [2002] refers to the Erlangen program in its title and is in some ways dazzling. However, it is not a text and I can't imagine using it as one; I recommend it only to individuals with some knowledge of group theory and some prior knowledge of geometries.

Brannan et al.'s first chapter is on conic sections. Though the preface states that knowledge of conic sections is assumed, the chapter is an exemplary and thorough treatment (as least with respect to rectangular coordinates). Besides being worthwhile in its own right, this section serves the additional purpose of preparing for treatments of conic sections in affine and projective geometries later.

Then follow treatments of affine geometry, projective geometry, inversive geometry (with some use of Möbius transformations), hyperbolic geometry, spherical geometry, and a last unifying chapter on the Kleinian view. Again, a knowledge of group theory is not really necessary, but a knowledge of linear algebra is. The treatment is accessible to well-prepared sophomores and is of interest to graduate students and professors.

Farin and Hansford's *The Geometry Toolbox for Graphics and Modeling* (GTGM) is a valuable book in several respects. First off, it is the answer to the question: What do I need to know to really get into computer graphics? While it is technically possible to get into computer graphics with less, it is not advisable.

Although the book touches upon other topics, it can be thought of as primarily about linear algebra and affine transformations; although not ostensibly written as a text on linear algebra, it can be used in that way. It does a great job of illustrating concepts such as parametric coordinates, eigenvalues, symmetric matrices, and so on. It could serve as a valuable secondary book for any student taking linear algebra out of a more traditional text. This book is written as a text, and it not only has better examples than most texts on linear algebra but sometimes clearer proofs while concentrating on the meat of linear algebra. Here it bears comparison with a more traditional text that at the same time tries a similar approach, namely, Banchoff and Werner [1992]. This latter text has a more traditional organization and is more complete with, for example, a fuller more cohesive treatment of determinants. It is a good book and it may well be a better linear algebra text than GTGM. But I like GTGM better; it is a better secondary book for the student and excellent in its stated goal of introducing graphics and modeling.

GTGM also gives a lot of attention to vectors and vector operations, as well as to affine transformations. Consider the problem of finding the distance from a point to a plane. The first edition of Strang [1993–2003], one of the best two introductions to linear algebra, featured in its first chapter precisely this problem. Apparently, Prof. Strang too discovered what I discovered in the classroom using his book: Ordinary sophomores—not his students at MIT, but users of the text elsewhere—seem to have trouble with these models and formulae. In the second edition [1998], he left this material out completely. Nonetheless, this material is more than germane to computer graphics (Banchoff and Wermer cover this material as well). GTGM does not make computer graphics easy or trivial, rather it makes it as easy as it is possible to make it (and this means a thorough grounding in linear algebra). For, you see (and you can quote me on this), . . . there is no royal road to computer graphics.

Lastly, GTGM lays the foundations for computer graphics (and some other modeling) but does not give much in the way of advanced techniques. For that, you can look to Buss [2003] which is better than most. A book of assorted tools on the subject, which I like a lot, is Glassner [1990].

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Mlodinow, Leonard. *Euclid's Window: The Story of Geometry from Parallel Lines to Hyperspace*. New York, NY: Touchstone, 2002; xii + 306, $14. ISBN 0–684–86524–6.

This is a book for the lay reader; and with 306 pp in a small format, it is a quick read. The book is strong on history, historical context, and recent physics. I like it because it shows the importance of Euclid—not the most brilliant of the Greek mathematicians, but he was a genius. His summary of all of mathematics to that time, *The Elements*, is the basis for all of Western mathematics and a major component of the foundations of Western science.

However, the reader will not learn much geometry per se. There are a great many endnotes, with plausible assertions, but others more knowledgeable than I have questions about their sources. On p. 27, we find

> Georg Cantor did groundbreaking work . . . his former teacher, a crab named Leopold Kronecker who opposed the irrationals, violently disagreed with Cantor and sabotaged his career at every turn. Cantor, unable to tolerate this, had a breakdown and spent his last days in a mental institution.

This is the story pushed by E.T. Bell [1968]. However, Cantor died a paranoid psychotic, a condition viewed today as almost entirely a biological disease. Moreover, Cantor was successful by almost any measure, and he had powerful friends as well as enemies.

This is indeed a book that undergraduates can read and enjoy. More than 30 years ago, when I was a sophomore, I swiped a book from my father's library; and that book influenced me more than any other towards a career in the mathematical sciences. The book was not a quick read, but it held my attention as much as anything that I had read until then. Compared to that book (Kline [1953]), the book reviewed here seems insubstantial.

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Hartshorne, Robin. 2000. *Geometry: Euclid and Beyond*. New York, NY: Springer-Verlag; xi + 526, $49.95. ISBN 0–387–98650–2.

My review [Cargal 2002] of Cederberg [2001] lists several first-rate books on traditional hyperbolic geometry; Greenberg [1997] is arguably the flagship.

This new text is by an eminent algebraic geometer who appears to relied on Greenberg as his number-one reader and advisor. Even though I have taught a course based on Greenberg [1997], I do not know how to compare the two books as *textbooks*, except to say that Hartshorne's is broader and more advanced. It has an emphasis on history, both Euclidean and nineteenth-century (hyperbolic geometry, Hilbert's axioms, and field theory). This book is both a great resource and a joy to read, a masterpiece and a classic in the making.

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Adams, Colin, Abigail Thompson, and Joel Hass. 2001. *How to Ace the Rest of Calculus: The Streetwise Guide*. New York: W.H. Freeman; ix + 272 + xvi, $14.96. ISBN 0–7167–4174–1.

Every student of calculus should have a book on the side of the *Calculus for Fools* type (any more plausible title is probably already in use). There is a large variety of them, almost all competent, and most worthwhile. A new edition of a classic example, *Calculus Made Easy*, has recently come out [Thompson 1998]. Also, one of our regular reviewers published *Calculus Demystified* [Krantz 2002]. Also receiving a great deal of attention is *How to Ace Calculus: The Streetwise Guide* [Adams et al. 1998], which seems as good as any; but now the same authors have come up with a second volume for "the rest of calculus."

Although there are plenty other books to help students through Calculus III, this is the first one I know of that compares to and extends the many books designed to help students through Calculus I (typically, they end before or around the middle of Calculus II). Since I am teaching Calculus III this semester, I can testify that the book complements my chosen text well. Calculus students need to know about this book.

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