Predicting Characters for Symmetry Operations with a Given Basis Set

The rules so far:

- Unchanged $\Rightarrow +1$ to character
- Changes sign $\Rightarrow -1$ to character (i.e. rotated 180°)
- Switches atom $\Rightarrow 0$ to character

Now consider $C_n$ rotations...

![Diagram of $C_3$ rotation]

Because the rotated basis vectors ($x'$ and $y'$) can be written as non-trivial linear combinations of the original basis vectors ($x$ and $y$), the above rules fail.

So we must work out matrix representatives:

$x' = -\sin 30° x - \cos 30° y + 0 z = -\frac{1}{2} x - \frac{\sqrt{3}}{2} y + 0 z$

$y' = \cos 30° x - \sin 30° y + 0 z = \frac{\sqrt{3}}{2} x - \frac{1}{2} y + 0 z$

$z' = 0 x + 0 y + z$

Or

\[
\begin{pmatrix}
-\frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\
\frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
(x) \\
(y) \\
(z)
\end{pmatrix}
= 
\begin{pmatrix}
x' \\
y' \\
z'
\end{pmatrix}
\]

$X = \text{sum of diagonal elements}$

$= 0$

(And vectors on the oxygens also give 0)

Q. What is $\chi(S_3)$?